

Specification September 2007

GCE Mathematics

Edexcel Advanced Subsidiary GCE in Mathematics (8371)
Edexcel Advanced Subsidiary GCE in Further Mathematics (8372)
Edexcel Advanced Subsidiary GCE in Pure Mathematics (8373)
Edexcel Advanced Subsidiary GCE in Further Mathematics
(Additional) (8374)
First examination 2009

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Edexcel GCE in Mathematics e-Spec

Your free e-Spec

Here is your free e-Spec, Edexcel's electronic version of the specification that will help you evaluate the content.

Everything you need

The e-Spec provides a range of useful resources including:

- A Senior Examiner explaining the changes to the new specification
- A customisable student guide to help recruit students
- A course planner to make it easy to plan delivery
- Links to sample assessment materials so you can see what is expected
- Information on the products and services provided by Edexcel to support the specification.

Easy-to-use

Just click on the walkthrough to see how easy and useful the e-Spec is and get more out of this specification today.

About this specification

Edexcel GCE in Mathematics, Further Mathematics, Pure Mathematics and Further Mathematics (Additional) is designed for use in school and colleges. It is part of a suite of GCE qualifications offered by Edexcel.

Key features of the specification

Edexcel's Mathematics specification has:

- all units equally weighted, allowing many different combinations of units and greater flexibility
- pathways leading to full Advanced Subsidiary and Advanced GCE in Mathematics, Further Mathematics, Pure Mathematics and Further Mathematics (Additional)
- updated Decision Mathematics 1 and Decision Mathematics 2 units, for a more balanced approach to content
- updated Further Pure Mathematics 1 unit for teaching in the first year of study
- updated Further Pure Mathematics 2 and Further Pure Mathematics 3 units to allow a coherent curriculum in further mathematics
- no change to Core, Mechanics or Statistics units content
- substantial Professional development and training programme
- past papers, specimen papers, examiner reports and further support materials available
- 18 units tested fully by written examination
- variety of endorsed electronic support material, including Exam Wizard, Topic Tutor and Exam Tutor
- endorsed textbooks and revision books, plus information available on how to map updated units to current textbooks.

Supporting you

Edexcel aims to provide the most comprehensive support for our qualifications. We have therefore published our own dedicated suite of resources for teachers and students written by qualification experts. We also endorse a wide range of materials from other publishers to give you a choice of approach.

For more information on our wide range of support and services for this GCE in Mathematics, Further Mathematics, Pure Mathematics and Further Mathematics (Additional) qualification, visit our GCE website: www.edexcel.org.uk/gce2008.

Specification updates

This specification is Issue 1 and is valid for Advanced Subsidiary (AS) examination from 2009 and A2 examination from 2009. If there are any significant changes to the specification Edexcel will write to centres to let them know. Changes will also be posted on our website.

For more information please visit www.edexcel.org.uk or www.edexcel.org.uk/gce2008.

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A Specification at a glance

Students study a variety of units, following pathways to their desired qualification.

Students may study units leading to the following awards:

- Advanced GCE in Mathematics
- Advanced GCE in Further Mathematics
- Advanced GCE in Pure Mathematics
- Advanced GCE In Further Mathematics (Additional)
- Advanced Subsidiary GCE in Mathematics
- Advanced Subsidiary GCE in Further Mathematics
- Advanced Subsidiary GCE in Pure Mathematics
- Advanced Subsidiary GCE in Further Mathematics (Additional).

Summary of unit content

Core Mathematics

Unit	Summary of unit content
C1	Algebra and functions; coordinate geometry in the (x, y) plane; sequences and series; differentiation; integration.
C2	Algebra and functions; coordinate geometry in the (x, y) plane; sequences and series; trigonometry; exponentials and logarithms; differentiation; integration.
C3	Algebra and functions; trigonometry; exponentials and logarithms; differentiation; numerical methods.
C4	Algebra and functions; coordinate geometry in the (x, y) plane; sequences and series; differentiation; integration; vectors.

Further Pure Mathematics

Unit	Summary of unit content
FP1	Series; complex numbers; numerical solution of equations; coordinate systems, matrix algebra, proof.
FP2	Inequalities; series, first order differential equations; second order differential equations; further complex numbers, Maclaurin and Taylor series.
FP3	Further matrix algebra; vectors, hyperbolic functions; differentiation; integration, further coordinate systems.

Mechanics

Unit	Summary of unit content
M1	Mathematical models in mechanics; vectors in mechanics; kinematics of a particle moving in a straight line; dynamics of a particle moving in a straight line or plane; statics of a particle; moments.
M2	Kinematics of a particle moving in a straight line or plane; centres of mass; work and energy; collisions; statics of rigid bodies.
M3	Further kinematics; elastic strings and springs; further dynamics; motion in a circle; statics of rigid bodies.
M4	Relative motion; elastic collisions in two dimensions; further motion of particles in one dimension; stability.
M5	Applications of vectors in mechanics; variable mass; moments of inertia of a rigid body; rotation of a rigid body about a fixed smooth axis.

Statistics

Unit	Summary of unit content
S1	Mathematical models in probability and statistics; representation and summary of data; probability; correlation and regression; discrete random variables; discrete distributions; the Normal distribution.
S2	The Binomial and Poisson distributions; continuous random variables; continuous distributions; samples; hypothesis tests.
S3	Combinations of random variables; sampling; estimation, confidence intervals and tests; goodness of fit and contingency tables; regression and correlation.
S4	Quality of tests and estimators; one-sample procedures; two-sample procedures.

Decision Mathematics

Unit	Summary of unit content
D1	Algorithms; algorithms on graphs; the route inspection problem; critical path analysis; linear programming; matchings.
D2	Transportation problems; allocation (assignment) problems; the travelling salesman; game theory; further linear programming, dynamic programming; flows in networks.

B Specification overview

Summary of awards

Advanced Subsidiary

Advanced subsidiary awards comprise three teaching units per award.

National classification code**	Cash-in code**	Award	Compulsory units	Optional units
2210	8371	GCE AS Mathematics	C1 and C2	M1, S1 or D1
2230	8372	GCE AS Further Mathematics	FP1	Any *
2230	8373	GCE AS Pure Mathematics	C1, C2, C3	
2230	8374	GCE AS Further Mathematics (Additional)	Any which have not been used for a previous qualification	

*For GCE AS Further Mathematics, excluded units are C1, C2, C3, C4

**See *Appendix E* for description of this code and all other codes relevant to this qualification

Advanced Subsidiary combinations

Combinations leading to an award in Advanced Subsidiary Mathematics comprise three AS units. Combinations leading to an award in Advanced Subsidiary Further Mathematics comprise three units. The combination leading to an award in Advanced Subsidiary Pure Mathematics comprises units C1, C2 and C3.

8371 Advanced Subsidiary Mathematics

Core Mathematics units C1 and C2 plus one of the Applications units M1, S1 or D1.

8372 Advanced Subsidiary Further Mathematics

Further Pure Mathematics unit FP1 plus two other units (excluding C1–C4). Students who are awarded certificates in both Advanced GCE Mathematics and AS Further Mathematics must use unit results from nine different teaching modules.

8373 Advanced Subsidiary Pure Mathematics

Core Mathematics units C1, C2 and C3.

8374 Additional Qualification in Advanced Subsidiary Further Mathematics (Additional)

Students who complete fifteen units will have achieved the equivalent of the standard of Advanced Subsidiary GCE Further Mathematics in their additional units. Such students will be eligible for the award of Advanced Subsidiary GCE Further Mathematics (Additional) in addition to the awards of Advanced GCE Mathematics and Advanced GCE Further Mathematics.

Advanced Level

Advanced level awards comprise six teaching units per award.

National classification code**	Cash-in code**	Qualifications	Compulsory units	Optional units
2210	9371	GCE Mathematics	C1, C2, C3, C4	M1 and S1 or M1 and D1 or S1 and D1 or S1 and S2 or M1 and M2 or D1 and D2
2230	9372	GCE Further Mathematics	FP1 and either FP2 or FP3	Any *
2230	9373	GCE Pure Mathematics	C1, C2, C3, C4, FP1	FP2 or FP3
2230	9374	GCE Further Mathematics (Additional)	Any which have not been used for a previous qualification	

*For GCE Further Mathematics, excluded units are C1, C2, C3, C4

** See *Appendix E* for description of this code and all other codes relevant to these qualifications.

Advanced GCE combinations

Combinations leading to an award in mathematics must comprise six units, including at least two A2 units. Combinations leading to an award in Further Mathematics must comprise six units, including at least three A2 units. Combinations leading to an award in Pure Mathematics comprise units C1–C4, FP1 and one of FP2 or FP3.

9371 Advanced GCE Mathematics

Core Mathematics units C1, C2, C3 and C4 plus two Applications units from the following six combinations: M1 and M2; S1 and S2; D1 and D2; M1 and S1; S1 and D1; M1 and D1.

9372 Advanced GCE Further Mathematics

Further Pure Mathematics units FP1, FP2, FP3 and a further three Applications units (excluding C1–C4) to make a total of six units; **or** FP1, either FP2 or FP3 and a further four Applications units (excluding C1–C4) to make a total of six units. Students who are awarded certificates in both Advanced GCE Mathematics and Advanced GCE Further Mathematics must use unit results from 12 different teaching modules.

9373 Advanced GCE Pure Mathematics

Core Mathematics units C1, C2, C3, C4, FP1, and either FP2 or FP3.

9374 Additional Qualification in Advanced Further Mathematics (Additional)

Students who complete eighteen units will have achieved the equivalent of the standard of Advanced GCE Further Mathematics (Additional) in their additional units. Such students will be eligible for the award of Advanced GCE Further Mathematics (Additional) in addition to the awards of Advanced GCE Mathematics and Advanced GCE Further Mathematics.

Summary of assessment requirements

Unit number	Unit title	Unit code*	Level	Method of assessment	Availability	First assessment	AS weighting	GCE weighting
C1	Core Mathematics 1	6663	AS	1 written paper	January and June	January 2009	33.3% of AS	16.67% of Advanced GCE
C2	Core Mathematics 2	6664	AS	1 written paper	January and June	January 2009	33.3% of AS	16.67% of Advanced GCE
C3	Core Mathematics 3	6665	A2	1 written paper	January and June	January 2009	33.3% of AS	16.67% of Advanced GCE
C4	Core Mathematics 4	6666	A2	1 written paper	January and June	January 2009	33.3% of AS	16.67% of Advanced GCE
FP1	Further Pure Mathematics 1	6667	AS	1 written paper	January and June	January 2009	33.3% of AS	16.67% of Advanced GCE
FP2	Further Pure Mathematics 2	6668	A2	1 written paper	June	June 2009	33.3% of AS	16.67% of Advanced GCE
FP3	Further Pure Mathematics 3	6669	A2	1 written paper	June	June 2009	33.3% of AS	16.67% of Advanced GCE
M1	Mechanics 1	6677	AS	1 written paper	January and June	January 2009	33.3% of AS	16.67% of Advanced GCE
M2	Mechanics 2	6678	A2	1 written paper	January and June	January 2009	33.3% of AS	16.67% of Advanced GCE
M3	Mechanics 3	6679	A2	1 written paper	January and June	January 2009	33.3% of AS	16.67% of Advanced GCE
M4	Mechanics 4	6680	A2	1 written paper	June	June 2009	33.3% of AS	16.67% of Advanced GCE
M5	Mechanics 5	6681	A2	1 written paper	June	June 2009	33.3% of AS	16.67% of Advanced GCE

Unit number	Unit title	Unit code*	Level	Method of assessment	Availability	First assessment	AS weighting	GCE weighting
S1	Statistics 1	6683	AS	1 written paper	January and June	January 2009	33.3% of AS	16.67% of Advanced GCE
S2	Statistics 2	6684	A2	1 written paper	January and June	January 2009	33.3% of AS	16.67% of Advanced GCE
S3	Statistics 3	6691	A2	1 written paper	June	June 2009	33.3% of AS	16.67% of Advanced GCE
S4	Statistics 4	6686	A2	1 written paper	June	June 2009	33.3% of AS	16.67% of Advanced GCE
D1	Decision Mathematics 1	6689	AS	1 written paper	January and June	January 2009	33.3% of AS	16.67% of Advanced GCE
D2	Decision Mathematics 2	6690	A2	1 written paper	June	June 2009	33.3% of AS	16.67% of Advanced GCE

*See *Appendix E* for a description of this code and all other codes relevant to this qualification.

- All examination papers last 1 hour 30 minutes
- All examination papers have 75 marks.
- C1 is a non-calculator paper: for all other unit examinations, calculators can be used.

Assessment objectives and weightings

	The assessment will test students' ability to:	Minimum weighting
AO1	recall, select and use their knowledge of mathematical facts, concepts and techniques in a variety of contexts	30%
AO2	construct rigorous mathematical arguments and proofs through use of precise statements, logical deduction and inference and by the manipulation of mathematical expressions, including the construction of extended arguments for handling substantial problems presented in unstructured form	30%
AO3	recall, select and use their knowledge of standard mathematical models to represent situations in the real world; recognise and understand given representations involving standard models; present and interpret results from such models in terms of the original situation, including discussion of the assumptions made and refinement of such models	10%
AO4	comprehend translations of common realistic contexts into mathematics; use the results of calculations to make predictions, or comment on the context; and, where appropriate, read critically and comprehend longer mathematical arguments or examples of applications	5%
AO5	use contemporary calculator technology and other permitted resources (such as formulae booklets or statistical tables) accurately and efficiently; understand when not to use such technology, and its limitations. Give answers to appropriate accuracy.	5%

Relationship of assessment objectives to units

All figures in the following table are expressed as marks out of 75.

Assessment objective	A01	A02	A03	A04	A05
Core Mathematics C1	30–35	25–30	5–15	5–10	1–5
Core Mathematics C2	25–30	25–30	5–10	5–10	5–10
Core Mathematics C3	25–30	25–30	5–10	5–10	5–10
Core Mathematics C4	25–30	25–30	5–10	5–10	5–10
Further Pure Mathematics FP1	25–30	25–30	0–5	5–10	5–10
Further Pure Mathematics FP2	25–30	25–30	0–5	7–12	5–10
Further Pure Mathematics FP3	25–30	25–30	0–5	7–12	5–10
Mechanics M1	20–25	20–25	15–20	6–11	4–9
Mechanics M2	20–25	20–25	10–15	7–12	5–10
Mechanics M3	20–25	25–30	10–15	5–10	5–10
Mechanics M4	20–25	20–25	15–20	5–10	5–10
Mechanics M5	20–25	20–25	15–20	5–10	5–10
Statistics S1	20–25	20–25	15–20	5–10	5–10
Statistics S2	25–30	20–25	10–15	5–10	5–10
Statistics S3	25–30	20–25	10–15	5–10	5–10
Statistics S4	15–20	15–20	15–20	5–10	10–15
Decision Mathematics D1	20–25	20–25	15–20	5–10	5–10
Decision Mathematics D2	25–30	20–25	10–15	7–13	0–5

Qualification summary

Subject criteria

The General Certificate of Education is part of the Level 3 provision. This specification is based on the GCE AS and A Level Subject criteria for Mathematics, which is prescribed by the regulatory authorities and is mandatory for all awarding bodies.

The GCE in Mathematics enables students to follow a flexible course in mathematics, to better tailor a course to suit the individual needs and goals.

Aims

The 18 units have been designed for schools and colleges to produce courses which will encourage students to:

- develop their understanding of mathematics and mathematical processes in a way that promotes confidence and fosters enjoyment
- develop abilities to reason logically and recognise incorrect reasoning, to generalise and to construct mathematical proofs
- extend their range of mathematical skills and techniques and use them in more difficult, unstructured problems
- develop an understanding of coherence and progression in mathematics and of how different areas of mathematics can be connected
- recognise how a situation may be represented mathematically and understand the relationship between 'real-world' problems and standard and other mathematical models and how these can be refined and improved
- use mathematics as an effective means of communication
- read and comprehend mathematical arguments and articles concerning applications of mathematics
- acquire the skills needed to use technology such as calculators and computers effectively, recognise when such use may be inappropriate and be aware of limitations
- develop an awareness of the relevance of mathematics to other fields of study, to the world of work and to society in general
- take increasing responsibility for their own learning and the evaluation of their own mathematical development.

AS/A2 knowledge and understanding and skills

The knowledge, understanding and skills required for all Mathematics specifications are contained in the subject core. The units C1, C2, C3 and C4 comprise this core material.

C**Mathematics, Further Mathematics, Pure Mathematics
and Further Mathematics (Additional) unit content**

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Unit C2 Core Mathematics 2	25
Unit C3 Core Mathematics 3	31
Unit C4 Core Mathematics 4	37
Unit FP1 Further Pure Mathematics 1	43
Unit FP2 Further Pure Mathematics 2	49
Unit FP3 Further Pure Mathematics 3	53
Unit M1 Mechanics 1	59
Unit M2 Mechanics 2	65
Unit M3 Mechanics 3	69
Unit M4 Mechanics 4	73
Unit M5 Mechanics 5	75
Unit S1 Statistics 1	79
Unit S2 Statistics 2	85
Unit S3 Statistics 3	89
Unit S4 Statistics 4	93
Unit D1 Decision Mathematics 1	97
Unit D2 Decision Mathematics 2	101

Structure of qualification

Compulsory unit	Optional unit
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GCE AS Mathematics

Core Mathematics 1	Core Mathematics 2	Application unit M1, S1 or D1
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GCE AS Further Mathematics

Further Pure Mathematics 1	Application or FP unit	Application or FP unit
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GCE AS Pure Mathematics

Core Mathematics 1	Core Mathematics 2	Core Mathematics 3
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GCE AS Further Mathematics (Additional)

Application or FP unit	Application or FP unit	Application unit
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GCE A Level Mathematics

Core Mathematics 1	Core Mathematics 2	Application unit M1, S1 or D1
Core Mathematics 3	Core Mathematics 4	Application unit M1, S1 or D1 or M2, S2 or D2

GCE A Level Further Mathematics

Further Pure Mathematics 1	Application or FP unit	Application unit
Further Pure Mathematics 2 or 3	Application unit	Application unit

GCE A Level Pure Mathematics

Core Mathematics 1	Core Mathematics 2	Core Mathematics 3
Core Mathematics 4	Further Pure Mathematics 1	Further Pure Mathematics 2 or 3

GCE A Level Further Mathematics (Additional)

Application or FP unit	Application unit	Application unit
Application unit	Application unit	Application unit

Unit C1 Core Mathematics 1

AS compulsory unit for GCE AS and GCE Mathematics, GCE AS and GCE Pure Mathematics

C1.1 Unit description

Algebra and functions; coordinate geometry in the (x, y) plane; sequences and series; differentiation; integration.

C1.2 Assessment information

Preamble

Construction and presentation of rigorous mathematical arguments through appropriate use of precise statements and logical deduction, involving correct use of symbols and appropriate connecting language is required. Students are expected to exhibit correct understanding and use of mathematical language and grammar in respect of terms such as 'equals', 'identically equals', 'therefore', 'because', 'implies', 'is implied by', 'necessary', 'sufficient', and notation such as \therefore , \Rightarrow , \Leftarrow and \Leftrightarrow .

Examination

The examination will consist of one 1½ hour paper. It will contain about ten questions of varying length. The mark allocations per question will be stated on the paper. All questions should be attempted.

For this unit, students may **not** have access to any calculating aids, including log tables and slide rules.

Formulae

Formulae which students are expected to know are given below and these will not appear in the booklet, *Mathematical Formulae including Statistical Formulae and Tables*, which will be provided for use with the paper. Questions will be set in SI units and other units in common usage.

This section lists formulae that students are expected to remember and that may not be included in formulae booklets.

Quadratic equations

$ax^2 + bx + c = 0$ has roots $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Differentiation

function

$$x^n$$

derivative

$$nx^{n-1}$$

Integration

function

$$x^n$$

integral

$$\frac{1}{n+1} x^{n+1} + c, n \neq -1$$

1 Algebra and functions

What students need to learn:

Laws of indices for all rational exponents.

The equivalence of $a^{m/n}$ and $\sqrt[n]{a^m}$ should be known.

Use and manipulation of surds.

Students should be able to rationalise denominators.

Quadratic functions and their graphs.

The discriminant of a quadratic function.

Completing the square. Solution of quadratic equations.

Solution of quadratic equations by factorisation, use of the formula and completing the square.

Simultaneous equations: analytical solution by substitution.

For example, where one equation is linear and one equation is quadratic.

Solution of linear and quadratic inequalities.

For example, $ax + b > cx + d$,
 $px^2 + qx + r \geq 0$, $px^2 + qx + r < ax + b$.

Algebraic manipulation of polynomials, including expanding brackets and collecting like terms, factorisation.

Graphs of functions; sketching curves defined by simple equations. Geometrical interpretation of algebraic solution of equations. Use of intersection points of graphs of functions to solve equations.

Knowledge of the effect of simple transformations on the graph of $y = f(x)$ as represented by $y = af(x)$, $y = f(x) + a$, $y = f(x + a)$, $y = f(ax)$.

Students should be able to use brackets. Factorisation of polynomials of degree n , $n \leq 3$, eg $x^3 + 4x^2 + 3x$. The notation $f(x)$ may be used. (Use of the factor theorem is *not* required.)

Functions to include simple cubic functions and the reciprocal function $y = \frac{k}{x}$ with $x \neq 0$.

Knowledge of the term asymptote is expected.

Students should be able to apply one of these transformations to any of the above functions (quadratics, cubics, reciprocal) and sketch the resulting graph.

Given the graph of any function $y = f(x)$ students should be able to sketch the graph resulting from one of these transformations.

2 Coordinate geometry in the (x, y) plane

What students need to learn:

Equation of a straight line, including the forms $y - y_1 = m(x - x_1)$ and $ax + by + c = 0$.

To include:

- (i) the equation of a line through two given points
- (ii) the equation of a line parallel (or perpendicular) to a given line through a given point. For example, the line perpendicular to the line $3x + 4y = 18$ through the point $(2, 3)$ has equation $y - 3 = \frac{4}{3}(x - 2)$.

Conditions for two straight lines to be parallel or perpendicular to each other.

3 Sequences and series

What students need to learn:

Sequences, including those given by a formula for the n th term and those generated by a simple relation of the form $x_{n+1} = f(x_n)$.

Arithmetic series, including the formula for the sum of the first n natural numbers.

The general term and the sum to n terms of the series are required. The proof of the sum formula should be known.

Understanding of Σ notation will be expected.

4 Differentiation

What students need to learn:

The derivative of $f(x)$ as the gradient of the tangent to the graph of $y = f(x)$ at a point; the gradient of the tangent as a limit; interpretation as a rate of change; second order derivatives.

Differentiation of x^n , and related sums and differences.

Applications of differentiation to gradients, tangents and normals.

For example, knowledge that $\frac{dy}{dx}$ is the rate of change of y with respect to x . Knowledge of the chain rule is not required.

The notation $f'(x)$ may be used.

For example, for $n \neq 1$, the ability to differentiate expressions such as $(2x + 5)(x - 1)$ and $\frac{x^2 + 5x - 3}{3x^{1/2}}$ is expected.

Use of differentiation to find equations of tangents and normals at specific points on a curve.

5 Integration

What students need to learn:

Indefinite integration as the reverse of differentiation.

Integration of x^n .

Students should know that a constant of integration is required.

For example, the ability to integrate expressions such as

$\frac{1}{2}x^2 - 3x^{-\frac{1}{2}}$ and $\frac{(x+2)^2}{x^{\frac{1}{2}}}$ is

expected.

Given $f'(x)$ and a point on the curve, students should be able to find an equation of the curve in the form $y = f(x)$.

Unit C2 Core Mathematics 2

AS compulsory unit for GCE AS and GCE Mathematics, GCE AS and GCE Pure Mathematics

C2.1 Unit description

Algebra and functions; coordinate geometry in the (x, y) plane; sequences and series; trigonometry; exponentials and logarithms; differentiation; integration.

C2.2 Assessment information

Prerequisites

A knowledge of the specification for C1, its preamble and its associated formulae, is assumed and may be tested.

Examination

The examination will consist of one 1½ hour paper. It will contain about nine questions of varying length. The mark allocations per question which will be stated on the paper. All questions should be attempted.

Calculators

Students are expected to have available a calculator with at least the following keys: $+$, $-$, \times , \div , π , x^2 , \sqrt{x} , $\frac{1}{x}$, x^y , $\ln x$, e^x , $x!$, sine, cosine and tangent and their inverses in degrees and decimals of a degree, and in radians; memory. Calculators with a facility for symbolic algebra, differentiation and/or integration are not permitted.

Formulae

Formulae which students are expected to know are given below and these will not appear in the booklet, *Mathematical Formulae including Statistical Formulae and Tables*, which will be provided for use with the paper. Questions will be set in SI units and other units in common usage.

This section lists formulae that students are expected to remember and that may not be included in formulae booklets.

Laws of logarithms

$$\log_a x + \log_a y = \log_a(xy)$$

$$\log_a x - \log_a y \equiv \log_a \left(\frac{x}{y} \right)$$

$$k \log_a x = \log_a(x^k)$$

Trigonometry

In the triangle ABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\text{area} = \frac{1}{2} ab \sin C$$

Area

$$\text{area under a curve} = \int_a^b y \, dx \quad (y \geq 0)$$

1 Algebra and functions

What students need to learn:

Simple algebraic division; use of the Factor Theorem and the Remainder Theorem.

Only division by $(x + a)$ or $(x - a)$ will be required.

Students should know that if $f(x) = 0$ when $x = a$, then $(x - a)$ is a factor of $f(x)$.

Students may be required to factorise cubic expressions such as $x^3 + 3x^2 - 4$ and $6x^3 + 11x^2 - x - 6$.

Students should be familiar with the terms 'quotient' and 'remainder' and be able to determine the remainder when the polynomial $f(x)$ is divided by $(ax + b)$.

2 Coordinate geometry in the (x, y) plane

What students need to learn:

Coordinate geometry of the circle using the equation of a circle in the form $(x - a)^2 + (y - b)^2 = r^2$ and including use of the following circle properties:

- (i) the angle in a semicircle is a right angle;
- (ii) the perpendicular from the centre to a chord bisects the chord;
- (iii) the perpendicularity of radius and tangent.

Students should be able to find the radius and the coordinates of the centre of the circle given the equation of the circle, and vice versa.

3 Sequences and series

What students need to learn:

The sum of a finite geometric series; the sum to infinity of a convergent geometric series, including the use of $|r| < 1$.

Binomial expansion of $(1 + x)^n$ for positive integer n .

The notations $n!$ and $\binom{n}{r}$.

The general term and the sum to n terms are required.

The proof of the sum formula should be known.

Expansion of $(a + bx)^n$ may be required.

4 Trigonometry

What students need to learn:

The sine and cosine rules, and the area of a triangle in the form $\frac{1}{2} ab \sin C$.

Radian measure, including use for arc length and area of sector.

Sine, cosine and tangent functions. Their graphs, symmetries and periodicity.

Knowledge and use of $\tan \theta = \frac{\sin \theta}{\cos \theta}$, and $\sin^2 \theta + \cos^2 \theta = 1$.

Use of the formulae $s = r\theta$ and $A = \frac{1}{2} r^2 \theta$ for a circle.

Knowledge of graphs of curves with equations such as

$y = 3 \sin x$, $y = \sin \left(x + \frac{\pi}{6} \right)$, $y = \sin 2x$ is expected.

Solution of simple trigonometric equations in a given interval.

Students should be able to solve equations such as

$$\sin\left(x + \frac{\pi}{2}\right) = \frac{3}{4} \text{ for } 0 < x < 2\pi,$$

$$\cos(x + 30^\circ) = \frac{1}{2} \text{ for } -180^\circ < x < 180^\circ,$$

$$\tan 2x = 1 \text{ for } 90^\circ < x < 270^\circ,$$

$$6 \cos^2 x + \sin x - 5 = 0, 0^\circ \leq x < 360,$$

$$\sin^2\left(x + \frac{\pi}{6}\right) = \frac{1}{2} \text{ for } -\pi \leq x < \pi.$$

5 Exponentials and logarithms

What students need to learn:

$y = a^x$ and its graph.

Laws of logarithms

To include

$$\log_a xy = \log_a x + \log_a y,$$

$$\log_a \frac{x}{y} = \log_a x - \log_a y,$$

$$\log_a x^k = k \log_a x,$$

$$\log_a \frac{1}{x} = -\log_a x,$$

$$\log_a a = 1$$

The solution of equations of the form $a^x = b$.

Students may use the change of base formula.

6 Differentiation

What students need to learn:

Applications of differentiation to maxima and minima and stationary points, increasing and decreasing functions.

The notation $f''(x)$ may be used for the second order derivative.

To include applications to curve sketching. Maxima and minima problems may be set in the context of a practical problem.

7 Integration

What students need to learn:

Evaluation of definite integrals.

Interpretation of the definite integral as the area under a curve.

Students will be expected to be able to evaluate the area of a region bounded by a curve and given straight lines.

Eg find the finite area bounded by the curve $y = 6x - x^2$ and the line $y = 2x$.

$\int_x dy$ will not be required.

Approximation of area under a curve using the trapezium rule.

For example,

evaluate $\int_0^1 \sqrt{2x+1} \, dx$

using the values of $\sqrt{2x+1}$ at $x = 0, 0.25, 0.5, 0.75$ and 1 .

Unit C3 Core Mathematics 3

A2 compulsory unit for GCE Mathematics and GCE Pure Mathematics Mathematics

C3.1 Unit description

Algebra and functions; trigonometry; exponentials and logarithms; differentiation; numerical methods.

C3.2 Assessment information

Prerequisites and preamble

Prerequisites

A knowledge of the specifications for C1 and C2, their preambles, prerequisites and associated formulae, is assumed and may be tested.

Preamble

Methods of proof, including proof by contradiction and disproof by counter-example, are required. At least one question on the paper will require the use of proof.

Examination

The examination will consist of one 1½ hour paper. It will contain about seven questions of varying length. The mark allocations per question which will be stated on the paper. All questions should be attempted.

Calculators

Students are expected to have available a calculator with at least the following keys: +, −, ×, ÷, π, x^2 , \sqrt{x} , $\frac{1}{x}$, x^y , $\ln x$, e^x , $x!$, sine, cosine and tangent and their inverses in degrees and decimals of a degree, and in radians; memory. Calculators with a facility for symbolic algebra, differentiation and/or integration are not permitted.

Formulae

Formulae which students are expected to know are given below and these will not appear in the booklet, *Mathematical Formulae including Statistical Formulae and Tables*, which will be provided for use with the paper. Questions will be set in SI units and other units in common usage.

This section lists formulae that students are expected to remember and that may not be included in formulae booklets.

Trigonometry

$$\cos^2 A + \sin^2 A \equiv 1$$

$$\sec^2 A \equiv 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A \equiv 1 + \cot^2 A$$

$$\sin 2A \equiv 2 \sin A \cos A$$

$$\cos 2A \equiv \cos^2 A - \sin^2 A$$

$$\tan 2A \equiv \frac{2 \tan A}{1 - \tan^2 A}$$

Differentiation

function

$$\sin kx$$

$$\cos kx$$

$$e^{kx}$$

$$\ln x$$

$$f(x) + g(x)$$

$$f(x) g(x)$$

$$f(g(x))$$

derivative

$$k \cos kx$$

$$-k \sin kx$$

$$ke^{kx}$$

$$\frac{1}{x}$$

$$f'(x) + g'(x)$$

$$f'(x) g(x) + f(x) g'(x)$$

$$f'(g(x)) g'(x)$$

1 Algebra and functions

What students need to learn:

Simplification of rational expressions including factorising and cancelling, and algebraic division.

Denominators of rational expressions will be linear or quadratic, eg $\frac{1}{ax+b}$,

$$\frac{ax+b}{px^2+qx+r}, \frac{x^3+1}{x^2-1}$$

Definition of a function. Domain and range of functions. Composition of functions. Inverse functions and their graphs.

The concept of a function as a one-one or many-one mapping from \mathbb{R} (or a subset of \mathbb{R}) to \mathbb{R} . The notation $f: x \mapsto$ and $f(x)$ will be used.

Students should know that fg will mean 'do g first, then f '.

Students should know that if f^{-1} exists, then $f^{-1}f(x) = ff^{-1}(x) = x$.

The modulus function.

Students should be able to sketch the graphs of $y = ax + b$ and the graphs of $y = f(x)$ and $y = f(x)$, given the graph of $y = f(x)$.

Combinations of the transformations $y = f(x)$ as represented by $y = af(x)$, $y = f(x) + a$, $y = f(x + a)$, $y = f(ax)$.

Students should be able to sketch the graph of, for example, $y = 2f(3x)$, $y = f(-x) + 1$, given the graph of $y = f(x)$ or the graph of, for example, $y = 3 + \sin 2x$,

$$y = -\cos\left(x + \frac{\pi}{4}\right).$$

The graph of $y = f(ax + b)$ will *not* be required.

2 Trigonometry

What students need to learn:

Knowledge of secant, cosecant and cotangent and of arcsin, arccos and arctan. Their relationships to sine, cosine and tangent. Understanding of their graphs and appropriate restricted domains.

Knowledge and use of $\sec^2 \theta = 1 + \tan^2 \theta$ and $\operatorname{cosec}^2 \theta = 1 + \cot^2 \theta$.

Knowledge and use of double angle formulae; use of formulae for $\sin(A \pm B)$, $\cos(A \pm B)$ and $\tan(A \pm B)$ and of expressions for $a \cos \theta + b \sin \theta$ in the equivalent forms of $r \cos(\theta \pm a)$ or $r \sin(\theta \pm a)$.

Angles measured in both degrees and radians.

To include application to half angles. Knowledge of the t ($\tan \frac{1}{2}\theta$) formulae will *not* be required.

Students should be able to solve equations such as

$a \cos \theta + b \sin \theta = c$ in a given interval, and to prove simple identities such as

$$\cos x \cos 2x + \sin x \sin 2x \equiv \cos x.$$

3 Exponentials and logarithms

What students need to learn:

The function e^x and its graph.

The function $\ln x$ and its graph; $\ln x$ as the inverse function of e^x .

To include the graph of $y = e^{ax+b} + c$.

Solution of equations of the form $e^{ax+b} = p$ and $\ln(ax+b) = q$ is expected.

4 Differentiation

What students need to learn:

Differentiation of e^x , $\ln x$, $\sin x$, $\cos x$, $\tan x$ and their sums and differences.

Differentiation using the product rule, the quotient rule and the chain rule.

The use of $\frac{dy}{dx} = \frac{1}{\left(\frac{dx}{dy}\right)}$.

Differentiation of $\operatorname{cosec} x$, $\cot x$ and $\sec x$ are required. Skill will be expected in the differentiation of functions generated from standard forms using products, quotients and composition, such as $2x^4 \sin x$, $\frac{e^{3x}}{x}$, $\cos x^2$ and $\tan^2 2x$.

Eg finding $\frac{dy}{dx}$ for $x = \sin 3y$.

5 Numerical methods

What students need to learn:

Location of roots of $f(x) = 0$ by considering changes of sign of $f(x)$ in an interval of x in which $f(x)$ is continuous.

Approximate solution of equations using simple iterative methods, including recurrence relations of the form $x_{n+1} = f(x_n)$.

Solution of equations by use of iterative procedures for which leads will be given.

Unit C4 Core Mathematics 4

A2 compulsory unit for GCE Mathematics and GCE Pure Mathematics

C4.1 Unit description

Algebra and functions; coordinate geometry in the (x, y) plane; sequences and series; differentiation; integration; vectors.

C4.2 Assessment information

Prerequisites A knowledge of the specifications for C1, C2 and C3 and their preambles, prerequisites and associated formulae, is assumed and may be tested.

Examination The examination will consist of one 1½ hour paper. It will contain about seven questions of varying length. The mark allocations per question which will be stated on the paper. All questions should be attempted.

Calculators Students are expected to have available a calculator with at least the following keys: +, −, ×, ÷, π , x^2 , \sqrt{x} , $\frac{1}{x}$, x^y , $\ln x$, e^x , $x!$, sine, cosine and tangent and their inverses in degrees and decimals of a degree, and in radians; memory. Calculators with a facility for symbolic algebra, differentiation and/or integration are not permitted.

Formulae Formulae which students are expected to know are given overleaf and these will not appear in the booklet, *Mathematical Formulae including Statistical Formulae and Tables*, which will be provided for use with the paper. Questions will be set in SI units and other units in common usage.

This section lists formulae that students are expected to remember and that may not be included in formulae booklets.

Integration

function

 $\cos kx$ $\sin kx$ e^{kx} $\frac{1}{x}$ $f'(x) + g'(x)$ $f'(g(x)) g'(x)$

integral

 $\frac{1}{k} \sin kx + c$ $-\frac{1}{k} \cos kx + c$ $\frac{1}{k} e^{kx} + c$ $\ln |x| + c, x \neq 0$ $f(x) + g(x) + c$ $f(g(x)) + c$ **Vectors**

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} \cdot \begin{pmatrix} a \\ b \\ c \end{pmatrix} = xa + yb + zc$$

1 Algebra and functions**What students need to learn:**

Rational functions. Partial fractions (denominators not more complicated than repeated linear terms).

Partial fractions to include denominators such as $(ax + b)(cx + d)(ex + f)$ and $(ax + b)(cx + d)^2$.

The degree of the numerator may equal or exceed the degree of the denominator. Applications to integration, differentiation and series expansions.

Quadratic factors in the denominator such as $(x^2 + a)$, $a > 0$, are *not* required.

2 Coordinate geometry in the (x, y) plane

What students need to learn:

Parametric equations of curves and conversion between Cartesian and parametric forms.

Students should be able to find the area under a curve given its parametric equations. Students will *not* be expected to sketch a curve from its parametric equations.

3 Sequences and series

What students need to learn:

Binomial series for any rational n .

For $|x| < \frac{b}{a}$, students should be able to obtain the expansion of $(ax + b)^n$, and the expansion of rational functions by decomposition into partial fractions.

4 Differentiation

What students need to learn:

Differentiation of simple functions defined implicitly or parametrically.

The finding of equations of tangents and normals to curves given parametrically or implicitly is required.

Exponential growth and decay.

Knowledge and use of the result

$$\frac{d}{dx} (a^x) = a^x \ln a \text{ is expected.}$$

Formation of simple differential equations.

Questions involving connected rates of change may be set.

5 Integration

What students need to learn:

Integration of e^x , $\frac{1}{x}$, $\sin x$, $\cos x$.

To include integration of standard functions such as $\sin 3x$, $\sec^2 2x$, $\tan x$, e^{5x} , $\frac{1}{2x}$.

Students should recognise integrals of the form

$$\int \frac{f'(x)}{f(x)} dx = \ln f(x) + c.$$

Students are expected to be able to use trigonometric identities to integrate, for example, $\sin^2 x$, $\tan^2 x$, $\cos^2 3x$.

Evaluation of volume of revolution.

$\pi \int y^2 dx$ is required, but *not*

$\pi \int x^2 dy$. Students should be able to find a volume of revolution, given parametric equations.

Simple cases of integration by substitution and integration by parts. These methods as the reverse processes of the chain and product rules respectively.

Except in the simplest of cases the substitution will be given.

The integral $\int \ln x dx$ is required.

More than one application of integration by parts may be required, for example $\int x^2 e^x dx$.

Simple cases of integration using partial fractions.

Integration of rational expressions such as those arising from partial fractions, eg $\frac{2}{3x+5}$, $\frac{3}{(x-1)^2}$.

Note that the integration of other rational expressions, such as $\frac{x}{x^2+5}$ and $\frac{2}{(2x-1)^4}$ is also required (see above paragraphs).

Analytical solution of simple first order differential equations with separable variables.

General and particular solutions will be required.

Numerical integration of functions.

Application of the trapezium rule to functions covered in C3 and C4. Use of increasing number of trapezia to improve accuracy and estimate error will be required. Questions will not require more than three iterations.

Simpson's Rule is *not* required.

6 Vectors

What students need to learn:

Vectors in two and three dimensions.

Magnitude of a vector.

Students should be able to find a unit vector in the direction of \mathbf{a} , and be familiar with $|\mathbf{a}|$.

Algebraic operations of vector addition and multiplication by scalars, and their geometrical interpretations.

Position vectors.

$$\vec{OB} - \vec{OA} = \vec{AB} = \mathbf{b} - \mathbf{a} .$$

The distance between two points.

The distance d between two points (x_1, y_1, z_1) and (x_2, y_2, z_2) is given by $d^2 = (x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2$.

Vector equations of lines.

To include the forms $\mathbf{r} = \mathbf{a} + t\mathbf{b}$ and $\mathbf{r} = \mathbf{c} + t(\mathbf{d} - \mathbf{c})$.

Intersection, or otherwise, of two lines.

The scalar product. Its use for calculating the angle between two lines.

Students should know that for

$$\vec{OA} = \mathbf{a} = a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k} \text{ and}$$

$$\vec{OB} = \mathbf{b} = b_1\mathbf{i} + b_2\mathbf{j} + b_3\mathbf{k} \text{ then}$$

$$\mathbf{a} \cdot \mathbf{b} = a_1b_1 + a_2b_2 + a_3b_3 \text{ and}$$

$$\cos \angle AOB = \frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}| |\mathbf{b}|} .$$

Students should know that if $\mathbf{a} \cdot \mathbf{b} = 0$, and that \mathbf{a} and \mathbf{b} are non-zero vectors, then \mathbf{a} and \mathbf{b} are perpendicular.

Unit FP1 Further Pure Mathematics 1

GCE AS and GCE Further Mathematics and GCE Pure Mathematics compulsory unit

FP1.1 Unit description

Series; complex numbers; numerical solution of equations;
Coordinate systems, matrix algebra, proof.

FP1.2 Assessment information

Prerequisites A knowledge of the specifications for C1 and C2, their prerequisites, preambles and associated formulae is assumed and may be tested.

Examination The examination will consist of one 1½ hour paper. It will contain about nine questions with varying mark allocations per question which will be stated on the paper. All questions may be attempted.

Questions will be set in SI units and other units in common usage.

Calculators Students are expected to have available a calculator with at least the following keys: +, −, ×, ÷, π, x^2 , \sqrt{x} , $\frac{1}{x}$, x^y , $\ln x$, e^x , $x!$, sine, cosine and tangent and their inverses in degrees and decimals of a degree, and in radians; memory. Calculators with a facility for symbolic algebra, differentiation and/or integration are not permitted.

1 Complex numbers

What students need to learn:

Definition of complex numbers in the form $a + ib$ and $r \cos \theta + i r \sin \theta$.

The meaning of conjugate, modulus, argument, real part, imaginary part and equality of complex numbers should be known.

Sum, product and quotient of complex numbers.

$$|z_1 z_2| = |z_1| |z_2|$$

Knowledge of the result $\arg(z_1 z_2) = \arg z_1 + \arg z_2$ is not required.

Geometrical representation of complex numbers in the Argand diagram.

Geometrical representation of sums, products and quotients of complex numbers.

Complex solutions of quadratic equations with real coefficients.

Conjugate complex roots of polynomial equations with real coefficients.

Knowledge that if z_1 is a root of $f(z) = 0$ then z_1^* is also a root.

2 Numerical solution of equations

What students need to learn:

Equations of the form $f(x) = 0$ solved numerically by:

- (i) interval bisection,
- (ii) linear interpolation,
- (iii) the Newton-Raphson process.

$f(x)$ will only involve functions used in C1 and C2.

For the Newton-Raphson process, the only differentiation required will be as defined in unit C1.

3 Coordinate systems

What students need to learn:

Cartesian equations for the parabola and rectangular hyperbola.

Students should be familiar with the equations:

$$y^2 = 4ax \text{ or } x = at^2, y = 2at \text{ and}$$

$$xy = c^2 \text{ or } x = ct, y = \frac{c}{t} .$$

Idea of parametric equation for parabola and rectangular hyperbola.

The idea of $(at^2, 2at)$ as a general point on the parabola is all that is required.

The focus-directrix property of the parabola.

Concept of focus and directrix and parabola as locus of points equidistant from focus and directrix.

Tangents and normals to these curves.

Differentiation of

$$y = 2a^{\frac{1}{2}}x^{\frac{1}{2}}, y = \frac{c^2}{x}.$$

Parametric differentiation is not required.

4 Matrix Algebra

What students need to learn:

Linear transformations of column vectors in two dimensions and their matrix representation.

The transformation represented by \mathbf{AB} is the transformation represented by \mathbf{B} followed by the transformation represented by \mathbf{A} .

Addition and subtraction of matrices.

Multiplication of a matrix by a scalar.

Products of matrices.

Evaluation of 2×2 determinants.

Singular and non-singular matrices.

Inverse of 2×2 matrices.

Use of the relation $(\mathbf{AB})^{-1} = \mathbf{B}^{-1}\mathbf{A}^{-1}$.

Combinations of transformations.

Applications of matrices to geometrical transformations.

Identification and use of the matrix representation of single and combined transformations from: reflection in coordinate axes and lines $y = \pm x$, rotation of multiples of 45° about $(0, 0)$ and enlargement about centre $(0, 0)$, with scale factor, $(k \neq 0)$, where $k \in \mathbb{R}$.

The inverse (when it exists) of a given transformation or combination of transformations.

Idea of the determinant as an area scale factor in transformations.

5 Series

What students need to learn:

Summation of simple finite series.

Students should be able to sum series such as

$$\sum_{r=1}^n r, \quad \sum_{r=1}^n r^2, \quad \sum_{r=1}^n r(r^2 + 2).$$

The method of differences is not required.

6 Proof

What students need to learn:

Proof by mathematical induction.

To include induction proofs for

(i) summation of series

eg show $\sum_{r=1}^n r^3 = \frac{1}{4}n^2(n+1)^2$ or

$$\sum_{r=1}^n r(r+1) = \frac{n(n+1)(n+2)}{3}$$

(ii) divisibility

eg show $3^{2n} + 11$ is divisible by 4.

(iii) finding general terms in a sequence

eg if $u_{n+1} = 3u_n + 4$ with

$u_1 = 1$, prove that $u_n = 3^n - 2$.

(iv) matrix products

eg show

$$\begin{pmatrix} -2 & -1 \\ 9 & 4 \end{pmatrix}^n = \begin{pmatrix} 1-3n & -n \\ 9n & 3n+1 \end{pmatrix}.$$

FP2.1 Unit description

Inequalities; series, first order differential equations; second order differential equations; further complex numbers, Maclaurin and Taylor series.

FP2.2 Assessment information

Prerequisites

A knowledge of the specifications for C1, C2, C3, C4 and FP1, their prerequisites, preambles and associated formulae is assumed and may be tested.

Examination

The examination will consist of one 1½ hour paper. It will contain about eight questions with varying mark allocations per question which will be stated on the paper. All questions may be attempted.

Questions will be set in SI units and other units in common usage.

Calculators

Students are expected to have available a calculator with at least the following keys: +, −, ×, ÷, π, x^2 , \sqrt{x} , $\frac{1}{x}$, x^y , $\ln x$, e^x , $x!$, sine, cosine and tangent and their inverses in degrees and decimals of a degree, and in radians; memory. Calculators with a facility for symbolic algebra, differentiation and/or integration are not permitted.

1 Inequalities

What students need to learn:

The manipulation and solution of algebraic inequalities and inequations, including those involving the modulus sign.

The solution of inequalities such as

$$\frac{1}{x-a} > \frac{x}{x-b} \quad |x^2 - 1| > 2(x+1).$$

2 Series

What students need to learn:

Summation of simple finite series using the method of differences.

Students should be able to sum series such as $\sum_{r=1}^n \frac{1}{r(r+1)}$ by using partial fractions such as

$$\frac{1}{r(r+1)} = \frac{1}{r} - \frac{1}{r+1}.$$

3 Further Complex Numbers

What students need to learn:

Euler's relation $e^{i\theta} = \cos \theta + i \sin \theta$.

Students should be familiar with

$$\cos \theta = \frac{1}{2}(e^{i\theta} + e^{-i\theta}) \text{ and}$$

$$\sin \theta = \frac{1}{2i}(e^{i\theta} - e^{-i\theta}).$$

De Moivre's theorem and its application to trigonometric identities and to roots of a complex number.

To include finding $\cos n\theta$ and $\sin m\theta$ in terms of powers of $\sin \theta$ and $\cos \theta$ and also powers of $\sin \theta$ and $\cos \theta$ in terms of multiple angles. Students should be able to prove De Moivre's theorem for any integer n .

Loci and regions in the Argand diagram.

Loci such as $|z - a| = b$,

$$|z - a| = k|z - b|,$$

$$\arg(z - a) = \beta, \arg \frac{z - a}{z - b} = \beta \text{ and}$$

regions such as $|z - a| \leq |z - b|$,

$$|z - a| \leq b.$$

Elementary transformations from the z -plane to the w -plane.

Transformations such as $w = z^2$ and

$$w = \frac{az + b}{cz + d}, \text{ where } a, b, c, d \in \mathbb{C}, \text{ may}$$

be set.

4 First Order Differential Equations

What students need to learn:

Further solution of first order differential equations with separable variables.

The formation of the differential equation may be required. Students will be expected to obtain particular solutions and also sketch members of the family of solution curves.

First order linear differential equations of the form $\frac{dy}{dx} + Py = Q$ where P and Q are functions of x .

The integrating factor $e^{\int P dx}$ may be quoted without proof.

Differential equations reducible to the above types by means of a given substitution.

5 Second Order Differential Equations

What students need to learn:

The linear second order differential equation $a\frac{d^2y}{dx^2} + b\frac{dy}{dx} + cy = f(x)$ where a , b and c are real constants and the particular integral can be found by inspection or trial.

The auxiliary equation may have real distinct, equal or complex roots. $f(x)$ will have one of the forms ke^{px} , $A + Bx$, $p + qx + cx^2$ or $m \cos \omega x + n \sin \omega x$.

Students should be familiar with the terms 'complementary function' and 'particular integral'.

Students should be able to solve equations of the form

$$\frac{d^2y}{dx^2} + 4y = \sin 2x.$$

Differential equations reducible to the above types by means of a given substitution.

6 Maclaurin and Taylor series

What students need to learn:

Third and higher order derivatives.

Derivation and use of Maclaurin series.

Derivation and use of Taylor series.

Use of Taylor series method for series solutions of differential equations.

The derivation of the series expansion of e^x , $\sin x$, $\cos x$, $\ln(1+x)$ and other simple functions may be required.

The derivation, for example, of the expansion of $\sin x$ in ascending powers of $(x-\pi)$ up to and including the term in $(x-\pi)^3$.

Students may, for example, be required to find the solution in powers of x as far as the term in x^4 , of the differential equation

$$\frac{d^2y}{dx^2} + x \frac{dy}{dx} + y = 0,$$

such that $y = 1$, $\frac{dy}{dx} = 0$ at $x = 0$.

7 Polar Coordinates

What students need to learn:

Polar coordinates (r, θ) , $r \geq 0$.

Use of the formula $\frac{1}{2} \int_{\alpha}^{\beta} r^2 d\theta$ for area.

The sketching of curves such as

$$\begin{aligned} \theta = \alpha, r = p \sec(\alpha - \theta), r = a, \\ r = 2a \cos \theta, r = k\theta, r = a(1 \pm \cos \theta), \\ r = a(3 + 2 \cos \theta), r = a \cos 2\theta \text{ and} \\ r^2 = a^2 \cos 2\theta \text{ may be set.} \end{aligned}$$

The ability to find tangents parallel to, or at right angles to, the initial line is expected.

Unit FP3 Further Pure Mathematics 3

GCE Further Mathematics GCE Pure Mathematics and Further Mathematics
(Additional) A2 optional unit

FP3.1 Unit description

Further matrix algebra; vectors, hyperbolic functions; differentiation; integration, further coordinate systems

FP3.3 Assessment information

Prerequisites A knowledge of the specifications for C1, C2, C3, C4 and FP1, their prerequisites, preambles and associated formulae is assumed and may be tested.

Examination The examination will consist of one 1½ hour paper. It will contain about eight questions with varying mark allocations per question which will be stated on the paper. All questions may be attempted.

Questions will be set in SI units and other units in common usage.

Calculators Students are expected to have available a calculator with at least the following keys: +, −, ×, ÷, π, x^2 , \sqrt{x} , $\frac{1}{x}$, x^y , $\ln x$, e^x , $x!$, sine, cosine and tangent and their inverses in degrees and decimals of a degree, and in radians; memory. Calculators with a facility for symbolic algebra, differentiation and/or integration are not permitted.

1 Hyperbolic functions

What students need to learn:

Definition of the six hyperbolic functions in terms of exponentials. Graphs and properties of the hyperbolic functions.

For example, $\cosh x = \frac{1}{2}(e^x + e^{-x})$,

$$\operatorname{sech} x = \frac{1}{\cosh x} = \frac{2}{e^x + e^{-x}}.$$

Students should be able to derive and use simple identities such as $\cosh^2 x - \sinh^2 x = 1$ and $\cosh^2 x + \sinh^2 x = \cosh 2x$ and to solve equations such as $a \cosh x + b \sinh x = c$.

Inverse hyperbolic functions, their graphs, properties and logarithmic equivalents.

Eg $\operatorname{arsinh} x = \ln[x + \sqrt{(1 + x^2)}]$.

Students may be required to prove this and similar results.

2 Further Coordinate Systems

What students need to learn:

Cartesian and parametric equations for the ellipse and hyperbola.

Extension of work from FP1.

Students should be familiar with the equations:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 ; x = a \cos t, y = b \sin t.$$

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 ; x = a \sec t, \\ y = b \tan t, x = a \cosh t, y = b \sinh t.$$

The focus-directrix properties of the ellipse and hyperbola, including the eccentricity.

For example, students should know that, for the ellipse, $b^2 = a^2(1 - e^2)$, the foci are $(ae, 0)$ and $(-ae, 0)$ and the equations of the directrices are $x = +\frac{a}{e}$ and $x = -\frac{a}{e}$.

Tangents and normals to these curves.

The condition for $y = mx + c$ to be a tangent to these curves is expected to be known.

Simple loci problems.

3 Differentiation

What students need to learn:

Differentiation of hyperbolic functions and expressions involving them.

For example, $\tanh 3x$, $x \sinh^2 x$,
 $\frac{\cosh 2x}{\sqrt{(x+1)}}$.

Differentiation of inverse functions, including trigonometric and hyperbolic functions.

For example, $\arcsin x + x\sqrt{(1-x^2)}$,
 $\frac{1}{2} \operatorname{artanh} x^2$.

4 Integration

What students need to learn:

Integration of hyperbolic functions and expressions involving them.

Integration of inverse trigonometric and hyperbolic functions.

Integration using hyperbolic and trigonometric substitutions.

Use of substitution for integrals involving quadratic surds.

The derivation and use of simple reduction formulae.

The calculation of arc length and the area of a surface of revolution.

For example,

$$\int \operatorname{arsinh} x \, dx, \int \arctan x \, dx.$$

To include the integrals of $1/(a^2 + x^2)$, $1/\sqrt{a^2 - x^2}$, $1/\sqrt{a^2 + x^2}$, $1/\sqrt{x^2 - a^2}$.

In more complicated cases, substitutions will be given.

Students should be able to derive formulae such as

$$nI_n = (n-1)I_{n-2}, \quad n \geq 2,$$

$$\text{for } I_n = \int_0^{\frac{\pi}{2}} \sin^n x \, dx,$$

$$I_{n+2} = \frac{2 \sin(n+1)x}{n+1} + I_n$$

$$\text{for } I_n = \int \frac{\sin nx}{\sin x} \, dx, \quad n > 0.$$

The equation of the curve may be given in cartesian or parametric form. Equations in polar form will not be set.

5 Vectors

What students need to learn:

The vector product $\mathbf{a} \times \mathbf{b}$ and the triple scalar product $\mathbf{a} \cdot \mathbf{b} \times \mathbf{c}$.

Use of vectors in problems involving points, lines and planes.

The equation of a line in the form $(\mathbf{r} - \mathbf{a}) \times \mathbf{b} = \mathbf{0}$.

The equation of a plane in the forms $\mathbf{r} \cdot \mathbf{n} = p$, $\mathbf{r} = \mathbf{a} + s\mathbf{b} + t\mathbf{c}$.

The interpretation of $|\mathbf{a} \times \mathbf{b}|$ as an area and $\mathbf{a} \cdot \mathbf{b} \times \mathbf{c}$ as a volume.

Students may be required to use equivalent cartesian forms also.

Applications to include

(i) distance from a point to a plane,

(ii) line of intersection of two planes,

(iii) shortest distance between two skew lines.

Students may be required to use equivalent cartesian forms also.

6 Further Matrix Algebra

What students need to learn:

Linear transformations of column vectors in two and three dimensions and their matrix representation.

Combination of transformations. Products of matrices.

Transpose of a matrix.

Evaluation of 3×3 determinants.

Inverse of 3×3 matrices.

The inverse (when it exists) of a given transformation or combination of transformations.

Eigenvalues and eigenvectors of 2×2 and 3×3 matrices.

Reduction of symmetric matrices to diagonal form.

Extension of work from FP1 to 3 dimensions.

The transformation represented by \mathbf{AB} is the transformation represented by \mathbf{B} followed by the transformation represented by \mathbf{A} .

Use of the relation $(\mathbf{AB})^T = \mathbf{B}^T \mathbf{A}^T$.

Singular and non-singular matrices.

Use of the relation $(\mathbf{AB})^{-1} = \mathbf{B}^{-1} \mathbf{A}^{-1}$.

Normalised vectors may be required.

Students should be able to find an orthogonal matrix \mathbf{P} such that $\mathbf{P}^T \mathbf{A} \mathbf{P}$ is diagonal.

Unit M1 Mechanics 1

GCE AS and GCE Mathematics, GCE AS and GCE Further Mathematics and GCE AS and GCE Further Mathematics (Additional) AS optional unit

M1.1 Unit description

Mathematical models in mechanics; vectors in mechanics; kinematics of a particle moving in a straight line; dynamics of a particle moving in a straight line or plane; statics of a particle; moments.

M1.2 Assessment information

Prerequisites

Students are expected to have a knowledge of C1, its preambles and associated formulae and of vectors in two dimensions.

Examination

The examination will consist of one 1½ hour paper. It will contain about seven questions with varying mark allocations per question which will be stated on the paper. All questions may be attempted.

Calculators

Students are expected to have available a calculator with at least the following keys: +, −, ×, ÷, π , x^2 , \sqrt{x} , $\frac{1}{x}$, x^y , $\ln x$, e^x , sine, cosine and tangent and their inverses in degrees and decimals of a degree, and in radians; memory. Calculators with a facility for symbolic algebra, differentiation and/or integration are not permitted.

Formulae

Students are expected to know any other formulae which might be required by the specification and which are not included in the booklet, *Mathematical Formulae including Statistical Formulae and Tables*, which will be provided for use with the paper. Questions will be set in SI units and other units in common usage.

Students will be expected to know and be able to recall and use the following formulae:

$$\text{Momentum} = mv$$

$$\text{Impulse} = mv - mu$$

For constant acceleration:

$$v = u + at$$

$$s = ut + \frac{1}{2}at^2$$

$$s = vt - \frac{1}{2}at^2$$

$$v^2 = u^2 + 2as$$

$$s = \frac{1}{2}(u + v)t$$

1 Mathematical Models in Mechanics

What students need to learn:

The basic ideas of mathematical modelling as applied in Mechanics.

Students should be familiar with the terms: particle, lamina, rigid body, rod (light, uniform, non-uniform), inextensible string, smooth and rough surface, light smooth pulley, bead, wire, peg. Students should be familiar with the assumptions made in using these models.

2 Vectors in Mechanics

What students need to learn:

Magnitude and direction of a vector. Resultant of vectors may also be required.

Students may be required to resolve a vector into two components or use a vector diagram. Questions may be set involving the unit vectors \mathbf{i} and \mathbf{j} .

Application of vectors to displacements, velocities, accelerations and forces in a plane.

Use of $\text{velocity} = \frac{\text{change of displacement}}{\text{time}}$

in the case of constant velocity, and of $\text{acceleration} = \frac{\text{change of velocity}}{\text{time}}$

in the case of constant acceleration, will be required.

3 Kinematics of a particle moving in a straight line

What students need to learn:

Motion in a straight line with constant acceleration.

Graphical solutions may be required, including displacement-time, velocity-time, speed-time and acceleration-time graphs. Knowledge and use of formulae for constant acceleration will be required.

4 Dynamics of a particle moving in a straight line or plane

What students need to learn:

The concept of a force. Newton's laws of motion.

Simple problems involving constant acceleration in scalar form or as a vector of the form $a\mathbf{i} + b\mathbf{j}$.

Simple applications including the motion of two connected particles.

Problems may include

- (i) the motion of two connected particles moving in a straight line or under gravity when the forces on each particle are constant; problems involving smooth fixed pulleys and/or pegs may be set;
- (ii) motion under a force which changes from one fixed value to another, eg a particle hitting the ground;
- (iii) motion directly up or down a smooth or rough inclined plane.

Momentum and impulse. The impulse-momentum principle. The principle of conservation of momentum applied to two particles colliding directly.

Knowledge of Newton's law of restitution is not required. Problems will be confined to those of a one-dimensional nature.

Coefficient of friction.

An understanding of $F = \mu R$ when a particle is moving.

5 Statics of a particle

What students need to learn:

Forces treated as vectors. Resolution of forces.

Equilibrium of a particle under coplanar forces.
Weight, normal reaction, tension and thrust, friction.

Coefficient of friction.

Only simple cases of the application of the conditions for equilibrium to uncomplicated systems will be required.

An understanding of $F \leq \mu R$ in a situation of equilibrium.

6 Moments

What students need to learn:

Moment of a force.

Simple problems involving coplanar parallel forces acting on a body and conditions for equilibrium in such situations.

Unit M2 Mechanics 2

GCE Mathematics, GCE AS and GCE Further Mathematics, GCE AS and GCE Further Mathematics (Additional) A2 optional unit

M2.1 Unit description

Kinematics of a particle moving in a straight line or plane; centres of mass; work and energy; collisions; statics of rigid bodies.

M2.2 Assessment information

Prerequisites

A knowledge of the specification for M1 and its prerequisites and associated formulae, together with a knowledge of algebra, trigonometry, differentiation and integration, as specified in C1, C2 and C3, is assumed and may be tested.

Examination

The examination will consist of one 1½ hour paper. It will contain about seven questions with varying mark allocations per question which will be stated on the paper. All questions may be attempted.

Calculators

Students are expected to have available a calculator with at least the following keys: +, −, ×, ÷, π, x^2 , \sqrt{x} , $\frac{1}{x}$, x^y , $\ln x$, e^x , sine, cosine and tangent and their inverses in degrees and decimals of a degree, and in radians; memory. Calculators with a facility for symbolic algebra, differentiation and/or integration are not permitted.

Formulae

Students are expected to know any other formulae which might be required by the specification and which are not included in the booklet, *Mathematical Formulae including Statistical Formulae and Tables*, which will be provided for use with the paper. Questions will be set in SI units and other units in common usage.

Students will be expected to know and be able to recall and use the following formulae:

$$\text{Kinetic energy} = \frac{1}{2}mv^2$$

$$\text{Potential energy} = mgh$$

1 Kinematics of a particle moving in a straight line or plane

What students need to learn:

Motion in a vertical plane with constant acceleration, eg under gravity.

Simple cases of motion of a projectile.

Velocity and acceleration when the displacement is a function of time.

Differentiation and integration of a vector with respect to time.

The setting up and solution of equations of the form $\frac{dx}{dt} = f(t)$ or $\frac{dv}{dt} = g(t)$ will be consistent with the level of calculus in C2.

For example, given that, $\mathbf{r} = t^2\mathbf{i} + t^{3/2}\mathbf{j}$, find $\dot{\mathbf{r}}$ and $\ddot{\mathbf{r}}$ at a given time.

2 Centres of mass

What students need to learn:

Centre of mass of a discrete mass distribution in one and two dimensions.

Centre of mass of uniform plane figures, and simple cases of composite plane figures.

The use of an axis of symmetry will be acceptable where appropriate. Use of integration is not required. Figures may include the shapes referred to in the formulae book. Results given in the formulae book may be quoted without proof.

Simple cases of equilibrium of a plane lamina.

The lamina may

- (i) be suspended from a fixed point;
- (ii) free to rotate about a fixed horizontal axis;
- (iii) be put on an inclined plane.

3 Work and energy

What students need to learn:

Kinetic and potential energy, work and power. The work-energy principle. The principle of conservation of mechanical energy.

Problems involving motion under a constant resistance and/or up and down an inclined plane may be set.

4 Collisions

What students need to learn:

Momentum as a vector. The impulse-momentum principle in vector form. Conservation of linear momentum.

Direct impact of elastic particles. Newton's law of restitution. Loss of mechanical energy due to impact.

Students will be expected to know and use the inequalities $0 \leq e \leq 1$ (where e is the coefficient of restitution).

Successive impacts of up to three particles or two particles and a smooth plane surface.

Collision with a plane surface will not involve oblique impact.

5 Statics of rigid bodies

What students need to learn:

Moment of a force.

Equilibrium of rigid bodies.

Problems involving parallel and non-parallel coplanar forces. Problems may include rods or ladders resting against smooth or rough vertical walls and on smooth or rough ground.

Unit M3 Mechanics 3

GCE AS and GCE Further Mathematics and GCE AS and GCE Further Mathematics (Additional) A2 optional unit

M3.1 Unit description

Further kinematics; elastic strings and springs; further dynamics; motion in a circle; statics of rigid bodies.

M3.2 Assessment information

Prerequisites

A knowledge of the specifications for M1 and M2 and their prerequisites and associated formulae, together with a knowledge of differentiation, integration and differential equations, as specified in C1, C2, C3 and C4, is assumed and may be tested.

Examination

The examination will consist of one 1½ hour paper. The paper will contain about seven questions with varying mark allocations per question which will be stated on the paper. All questions may be attempted.

Calculators

Students are expected to have available a calculator with at least the following keys: +, −, ×, ÷, π, x^2 , \sqrt{x} , $\frac{1}{x}$, x^y , $\ln x$, e^x , sine, cosine and tangent and their inverses in degrees and decimals of a degree, and in radians; memory. Calculators with a facility for symbolic algebra, differentiation and/or integration are not permitted.

Formulae

Students are expected to know any other formulae which might be required by the specification and which are not included in the booklet, *Mathematical Formulae including Statistical Formulae and Tables*, which will be provided for use with the paper. Questions will be set in SI units and other units in common usage.

Students will be expected to know and be able to recall and use the following formulae:

The tension in an elastic string = $\frac{\lambda x}{l}$

The energy stored in an elastic string = $\frac{\lambda x^2}{2l}$

For SHM:

$$\ddot{x} = -\omega^2 x,$$

$$x = a \cos \omega t \text{ or } x = a \sin \omega t,$$

$$v^2 = \omega^2(a^2 - x^2),$$

$$T = \frac{2\pi}{\omega}$$

1 Further kinematics**What students need to learn:**

Kinematics of a particle moving in a straight line when the acceleration is a function of the displacement (x), or time (t).

The setting up and solution of

equations where $\frac{dv}{dt} = f(t)$,

$$v \frac{dv}{dx} = f(x), \frac{dx}{dt} = f(x) \text{ or } \frac{dx}{dt} = f(t)$$

will be consistent with the level of calculus required in units C1, C2 and C3.

2 Elastic strings and springs

What students need to learn:

Elastic strings and springs. Hooke's law.

Energy stored in an elastic string or spring.

Simple problems using the work-energy principle involving kinetic energy, potential energy and elastic energy.

3 Further dynamics

What students need to learn:

Newton's laws of motion, for a particle moving in one dimension, when the applied force is variable.

Simple harmonic motion.

Oscillations of a particle attached to the end of an elastic string or spring.

The solution of the resulting equations will be consistent with the level of calculus in units C2 and C3. Problems may involve the law of gravitation, ie the inverse square law.

Proof that a particle moves with simple harmonic motion in a given situation may be required (ie showing that $\ddot{x} = -\omega^2 x$).

Geometric or calculus methods of solution will be acceptable. Students will be expected to be familiar with standard formulae, which may be quoted without proof.

Oscillations will be in the direction of the string or spring only.

4 Motion in a circle

What students need to learn:

Angular speed.

Radial acceleration in circular motion. The forms $r\omega^2$

and $\frac{v^2}{r}$ are required.

Uniform motion of a particle moving in a horizontal circle.

Problems involving the 'conical pendulum', an elastic string, motion on a banked surface, as well as other contexts, may be set.

Motion of a particle in a vertical circle.

5 Statics of rigid bodies

What students need to learn:

Centre of mass of uniform rigid bodies and simple composite bodies.

The use of integration and/or symmetry to determine the centre of mass of a uniform body will be required.

Simple cases of equilibrium of rigid bodies.

To include

- (i) suspension of a body from a fixed point,
- (ii) a rigid body placed on a horizontal or inclined plane.

Unit M4 Mechanics 4

GCE AS and GCE Further Mathematics and GCE AS and GCE Further Mathematics (Additional) A2 optional unit

M4.1 Unit description

Relative motion; elastic collisions in two dimensions; further motion of particles in one dimension; stability.

M4.2 Assessment information

Prerequisites

A knowledge of the specifications for M1, M2 and M3 and their prerequisites and associated formulae, together with a knowledge of the calculus covered in FP1 and $\int \frac{1}{a^2 + x^2} dx$, is assumed and may be tested.

Examination

The examination will consist of one 1½ hour paper. It will contain about seven questions with varying mark allocations per question which will be stated on the paper. All questions may be attempted.

Calculators

Students are expected to have available a calculator with at least the following keys: +, −, ×, ÷, π, x², √x, $\frac{1}{x}$, x^y, ln x, e^x, sine, cosine and tangent and their inverses in degrees and decimals of a degree, and in radians; memory. Calculators with a facility for symbolic algebra, differentiation and/or integration are not permitted.

Formulae

Students are expected to know any other formulae which might be required by the specification and which are not included in the booklet, *Mathematical Formulae including Statistical Formulae and Tables*, which will be provided for use with the paper. Questions will be set in SI units and other units in common usage.

Students will be expected to know and be able to recall and use the following formulae:

$${}_A \mathbf{v}_B = \mathbf{v}_A - \mathbf{v}_B$$

1 Relative motion

What students need to learn:

Relative motion of two particles, including relative displacement and relative velocity.

Problems may be set in vector form and may involve problems of interception or closest approach including the determination of course required for closest approach.

2 Elastic collisions in two dimensions

What students need to learn:

Oblique impact of smooth elastic spheres and a smooth sphere with a fixed surface.

3 Further motion of particles in one dimension

What students need to learn:

Resisted motion of a particle moving in a straight line.

The resisting forces may include the forms $a + bv$ and $a + bv^2$ where a and b are constants and v is the speed.

Damped and/or forced harmonic motion.

The damping to be proportional to the speed. Solution of the relevant differential equations will be expected.

4 Stability

What students need to learn:

Finding equilibrium positions of a system from consideration of its potential energy.

Positions of stable and unstable equilibrium of a system.

Unit M5 Mechanics 5

GCE Further Mathematics and GCE AS and GCE Further Mathematics (Additional)
A2 optional unit

M5.1 Unit description

Applications of vectors in mechanics; variable mass; moments of inertia of a rigid body; rotation of a rigid body about a fixed smooth axis.

M5.2 Assessment information

Prerequisites A knowledge of the specifications for M1, M2, M3 and M4 and their prerequisites and associated formulae, together with a knowledge of scalar and vector products, and of differential equations as specified in FP1, is assumed and may be tested.

Examination The examination will consist of one 1½ hour paper. It will contain about seven questions with varying mark allocations per question which will be stated on the paper. All questions may be attempted.

Calculators Students are expected to have available a calculator with at least the following keys: +, −, ×, ÷, π, x^2 , \sqrt{x} , $\frac{1}{x}$, x^y , $\ln x$, e^x , sine, cosine and tangent and their inverses in degrees and decimals of a degree, and in radians; memory. Calculators with a facility for symbolic algebra, differentiation and/or integration are not permitted.

Formulae

Students are expected to know any other formulae which might be required by the specification and which are not included in the booklet, *Mathematical Formulae including Statistical Formulae and Tables*, which will be provided for use with the paper. Questions will be set in SI units and other units in common usage.

Students will be expected to know and be able to recall and use the following formulae:

Work done by a constant force = $\mathbf{F} \cdot \mathbf{d}$

Angular momentum = $I\dot{\theta}$

Rotational kinetic energy = $\frac{1}{2}I\dot{\theta}^2$

$L = I\ddot{\theta}$

$$\int_{t_1}^{t_2} L \, dt = I\omega_2 - I\omega_1$$

For constant angular acceleration:

$$\omega_1 = \omega_0 + \alpha t$$

$$\omega_1^2 = \omega_0^2 + 2\alpha\theta$$

$$\theta = \omega_0 t + \frac{1}{2} \alpha t^2$$

$$\theta = \frac{(\omega_0 + \omega_1)}{2} t$$

1 Applications of vectors in mechanics

What students need to learn:

Solution of simple vector differential equations.

Vector differential equations such as

$$\frac{d\mathbf{v}}{dt} = k\mathbf{v},$$

$$\frac{d^2\mathbf{r}}{dt^2} + 2k\frac{d\mathbf{r}}{dt} + (k^2 + n^2)\mathbf{r} = \mathbf{f}(t)$$

where k and n are constants,

$$\frac{d\mathbf{r}}{dt} + 4\mathbf{r} = \mathbf{ie}'.$$

Use of an integrating factor may be required.

Work done by a constant force using a scalar product.

Moment of a force using a vector product.

The moment of a force \mathbf{F} about O is defined as $\mathbf{r} \times \mathbf{F}$, where \mathbf{r} is the position vector of the point of application of \mathbf{F} .

The analysis of simple systems of forces in three dimensions acting on a rigid body.

The reduction of a system of forces acting on a body to a single force, single couple or a couple and a force acting through a stated point.

2 Variable mass

What students need to learn:

Motion of a particle with varying mass.

Students may be required to derive an equation of motion from first principles by considering the change in momentum over a small time interval.

3 Moments of inertia of a rigid body

What students need to learn:

Moments of inertia and radius of gyration of standard and composite bodies.

Use of integration including the proof of the standard results given in the formulae booklet will be required.

The parallel and perpendicular axes theorems.

4 Rotation of a rigid body about a fixed smooth axis

What students need to learn:

Motion of a rigid body about a fixed smooth horizontal or vertical axis.

Use of conservation of energy will be required. Calculation of the force on the axis will be required.

Angular momentum.

Kinetic energy.

Conservation of angular momentum. The effect of an impulse on a rigid body which is free to rotate about a fixed axis.

Simple pendulum and compound pendulum.

Unit S1 Statistics 1

GCE AS and GCE Mathematics, GCE AS and GCE Further Mathematics and GCE AS and GCE Further Mathematics (Additional) AS optional unit

S1.1 Unit description

Mathematical models in probability and statistics; representation and summary of data; probability; correlation and regression; discrete random variables; discrete distributions; the Normal distribution.

S1.2 Assessment information

Examination

The examination will consist of one 1½ hour paper. The paper will contain about seven questions with varying mark allocations per question which will be stated on the paper. All questions may be attempted.

Calculators

Students are expected to have available a calculator with at least the following keys: +, −, ×, ÷, π , x^2 , \sqrt{x} , $\frac{1}{x}$, x^y , $\ln x$, e^x , $x!$, sine, cosine and tangent and their inverses in degrees and decimals of a degree, and in radians; memory. Calculators with a facility for symbolic algebra, differentiation and/or integration are not permitted.

Formulae

Students are expected to know formulae which might be required by the specification and which are not included in the booklet, Mathematical Formulae including Statistical Formulae and Tables, which will be provided for use with the paper. Questions will be set in SI units and other units in common usage.

Students will be expected to know and be able to recall and use the following formulae:

$$\text{Mean} = \bar{x} = \frac{\sum x}{n} \text{ or } \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{(\text{Variance})}$$

$$\text{Interquartile range} = \text{IQR} = Q_3 - Q_1$$

$$P(A') = 1 - P(A)$$

For independent events A and B ,

$$P(B | A) = P(B), P(A | B) = P(A),$$

$$P(A \cap B) = P(A) P(B)$$

$$E(aX + b) = aE(X) + b$$

$$\text{Var}(aX + b) = a^2 \text{Var}(X)$$

Cumulative distribution function for a discrete random variable:

$$F(x_0) = P(X \leq x_0) = \sum_{x \leq x_0} p(x)$$

$$\text{Standardised Normal Random Variable } Z = \frac{X - \mu}{\sigma}$$

where $X \sim N(\mu, \sigma^2)$

1 Mathematical models in probability and statistics**What students need to learn:**

The basic ideas of mathematical modelling as applied in probability and statistics.

2 Representation and summary of data

What students need to learn:

Histograms, stem and leaf diagrams, box plots.

Using histograms, stem and leaf diagrams and box plots to compare distributions.

Back-to-back stem and leaf diagrams may be required.

Drawing of histograms, stem and leaf diagrams or box plots will not be the direct focus of examination questions.

Measures of location — mean, median, mode.

Calculation of mean, mode and median, range and interquartile range will not be the direct focus of examination questions.

Students will be expected to draw simple inferences and give interpretations to measures of location and dispersion. Significance tests will not be expected.

Data may be discrete, continuous, grouped or ungrouped. Understanding and use of coding.

Measures of dispersion — variance, standard deviation, range and interpercentile ranges.

Simple interpolation may be required. Interpretation of measures of location and dispersion.

Skewness. Concepts of outliers.

Students may be asked to illustrate the location of outliers on a box plot. Any rule to identify outliers will be specified in the question.

3 Probability

What students need to learn:

Elementary probability.

Sample space. Exclusive and complementary events.
Conditional probability.

Understanding and use of

$$P(A') = 1 - P(A),$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B),$$

$$P(A \cap B) = P(A) P(B|A).$$

Independence of two events.

$$P(B|A) = P(B), P(A|B) = P(A),$$

$$P(A \cap B) = P(A) P(B).$$

Sum and product laws.

Use of tree diagrams and Venn diagrams. Sampling with and without replacement.

4 Correlation and regression

What students need to learn:

Scatter diagrams. Linear regression.

Calculation of the equation of a linear regression line using the method of least squares. Students may be required to draw this regression line on a scatter diagram.

Explanatory (independent) and response (dependent) variables. Applications and interpretations.

Use to make predictions within the range of values of the explanatory variable and the dangers of extrapolation. Derivations will not be required. Variables other than x and y may be used. Linear change of variable may be required.

The product moment correlation coefficient, its use, interpretation and limitations.

Derivations and tests of significance will not be required.

5 Discrete random variables

What students need to learn:

The concept of a discrete random variable.

The probability function and the cumulative distribution function for a discrete random variable.

Simple uses of the probability function $p(x)$ where $p(x) = P(X = x)$.
Use of the cumulative distribution function:

$$F(x_0) = P(X \leq x_0) = \sum_{x \leq x_0} p(x)$$

Mean and variance of a discrete random variable.

Use of $E(X)$, $E(X^2)$ for calculating the variance of X .

Knowledge and use of

$$E(aX + b) = aE(X) + b,$$

$$\text{Var}(aX + b) = a^2 \text{Var}(X).$$

The discrete uniform distribution.

The mean and variance of this distribution.

6 The Normal distribution

What students need to learn:

The Normal distribution including the mean, variance and use of tables of the cumulative distribution function.

Knowledge of the shape and the symmetry of the distribution is required. Knowledge of the probability density function is not required. Derivation of the mean, variance and cumulative distribution function is not required. Interpolation is not necessary. Questions may involve the solution of simultaneous equations.

S2.1 Unit description

The Binomial and Poisson distributions; continuous random variables; continuous distributions; samples; hypothesis tests.

S2.2 Assessment information

Prerequisites

A knowledge of the specification for S1 and its prerequisites and associated formulae, together with a knowledge of differentiation and integration of polynomials, binomial coefficients in connection with the binomial distribution and the evaluation of the exponential function is assumed and may be tested.

Examination

The examination will consist of one 1½ hour paper. The paper will contain about seven questions with varying mark allocations per question which will be stated on the paper. All questions may be attempted.

Calculators

Students are expected to have available a calculator with at least the following keys: +, −, ×, ÷, π , x^2 , \sqrt{x} , $\frac{1}{x}$, x^y , $\ln x$, e^x , $x!$, sine, cosine and tangent and their inverses in degrees and decimals of a degree, and in radians; memory. Calculators with a facility for symbolic algebra, differentiation and/or integration are not permitted.

Formulae

Students are expected to know any other formulae which might be required by the specification and which are not included in the booklet, *Mathematical Formulae including Statistical Formulae and Tables*, which will be provided for use with the paper. Questions will be set in SI units and other units in common usage.

Students will be expected to know and be able to recall and use the following formulae:

For the continuous random variable X having probability density function $f(x)$,

$$P(a < X \leq b) = \int_a^b f(x) \, dx.$$

$$f(x) = \frac{dF(x)}{dx}.$$

1 The Binomial and Poisson distributions**What students need to learn:**

The binomial and Poisson distributions.

Students will be expected to use these distributions to model a real-world situation and to comment critically on their appropriateness. Cumulative probabilities by calculation or by reference to tables.

Students will be expected to use the additive property of the Poisson distribution — eg if the number of events per minute $\sim \text{Po}(\lambda)$ then the number of events per 5 minutes $\sim \text{Po}(5\lambda)$.

The mean and variance of the binomial and Poisson distributions.

No derivations will be required.

The use of the Poisson distribution as an approximation to the binomial distribution.

2 Continuous random variables

What students need to learn:

The concept of a continuous random variable.

The probability density function and the cumulative distribution function for a continuous random variable.

Use of the probability density function $f(x)$, where

$$P(a < X \leq b) = \int_a^b f(x) \, dx.$$

Use of the cumulative distribution function

$$F(x_0) = P(X \leq x_0) = \int_{-\infty}^{x_0} f(x) \, dx.$$

The formulae used in defining $f(x)$ will be restricted to simple polynomials which may be expressed piecewise.

Relationship between density and distribution functions.

$$f(x) = \frac{dF(x)}{dx}$$

Mean and variance of continuous random variables.

Mode, median and quartiles of continuous random variables.

3 Continuous distributions

What students need to learn:

The continuous uniform (rectangular) distribution.

Including the derivation of the mean, variance and cumulative distribution function.

Use of the Normal distribution as an approximation to the binomial distribution and the Poisson distribution, with the application of the continuity correction.

4 Hypothesis tests

What students need to learn:

Population, census and sample. Sampling unit, sampling frame.

Students will be expected to know the advantages and disadvantages associated with a census and a sample survey.

Concepts of a statistic and its sampling distribution.

Concept and interpretation of a hypothesis test. Null and alternative hypotheses.

Use of hypothesis tests for refinement of mathematical models.

Critical region.

Use of a statistic as a test statistic.

One-tailed and two-tailed tests.

Hypothesis tests for the parameter p of a binomial distribution and for the mean of a Poisson distribution.

Students are expected to know how to use tables to carry out these tests. Questions may also be set not involving tabular values. Tests on sample proportion involving the normal approximation will not be set.

S3.1 Unit description

Combinations of random variables; sampling; estimation, confidence intervals and tests; goodness of fit and contingency tables; regression and correlation

S3.2 Assessment information

Prerequisites

A knowledge of the specifications for S1 and S2 and their prerequisites and associated formulae is assumed and may be tested.

Examination

The examination will consist of one 1½ hour paper. The paper will contain about seven questions with varying mark allocations per question which will be stated on the paper. All questions may be attempted.

Calculators

Students are expected to have available a calculator with at least the following keys: +, −, ×, ÷, π, x^2 , \sqrt{x} , $\frac{1}{x}$, x^y , ln x , e^x , $x!$, sine, cosine and tangent and their inverses in degrees and decimals of a degree, and in radians; memory. Calculators with a facility for symbolic algebra, differentiation and/or integration are not permitted.

Formulae

Students are expected to know any other formulae which might be required by the specification and which are not included in the booklet, *Mathematical Formulae including Statistical Formulae and Tables*, which will be provided for use with the paper. Questions will be set in SI units and other units in common usage.

Students will be expected to know and be able to recall and use the following formulae:

$aX \pm bY \sim N(a\mu_x \pm b\mu_y, a^2\sigma_x^2 + b^2\sigma_y^2)$ where X and Y are independent and $X \sim N(\mu_x, \sigma_x^2)$ and $Y \sim N(\mu_y, \sigma_y^2)$.

1 Combinations of random variables

What students need to learn:

Distribution of linear combinations of independent Normal random variables.

If $X \sim N(\mu_x, \sigma_x^2)$ and $Y \sim N(\mu_y, \sigma_y^2)$

independently, then

$$aX \pm bY \sim N(a\mu_x \pm b\mu_y, a^2\sigma_x^2 + b^2\sigma_y^2).$$

No proofs required.

2 Sampling

What students need to learn:

Methods for collecting data. Simple random sampling. Use of random numbers for sampling.

Other methods of sampling: stratified, systematic, quota.

The circumstances in which they might be used. Their advantages and disadvantages.

3 Estimation, confidence intervals and tests

What students need to learn:

Concepts of standard error, estimator, bias.

The sample mean, \bar{x} , and the sample variance,

$$s^2 = \frac{1}{n-1} \sum_{i=1}^n (x_i - \bar{x})^2, \text{ as}$$

unbiased estimates of the corresponding population parameters.

The distribution of the sample mean \bar{X} .

\bar{X} has mean μ and variance $\frac{\sigma^2}{n}$.

If $X \sim N(\mu, \sigma^2)$ then $\bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$.

No proofs required.

Concept of a confidence interval and its interpretation.

Confidence limits for a Normal mean, with variance known.

Hypothesis tests for the mean of a Normal distribution with variance known.

Use of Central Limit theorem to extend hypothesis tests and confidence intervals to samples from non-Normal distributions. Use of large sample results to extend to the case in which the variance is unknown.

Hypothesis test for the difference between the means of two Normal distributions with variances known.

Use of large sample results to extend to the case in which the population variances are unknown.

Link with hypothesis tests.

Students will be expected to know how to apply the Normal distribution and use the standard error and obtain confidence intervals for the mean, rather than be concerned with any theoretical derivations.

Use of $\frac{\bar{X} - \mu}{S/\sqrt{n}} \sim N(0, 1)$.

$\frac{\bar{X} - \mu}{S/\sqrt{n}}$ can be treated as $N(0, 1)$ when n is large. A knowledge of the t -distribution is not required.

Use of
$$\frac{(\bar{X} - \bar{Y}) - (\mu_x - \mu_y)}{\sqrt{\frac{\sigma_x^2}{n_x} + \frac{\sigma_y^2}{n_y}}} \sim N(0, 1)$$
.

Use of
$$\frac{(\bar{X} - \bar{Y}) - (\mu_x - \mu_y)}{\sqrt{\frac{S_x^2}{n_x} + \frac{S_y^2}{n_y}}} \sim N(0, 1)$$
.

A knowledge of the t -distribution is not required.

4 Goodness of fit and contingency tables

What students need to learn:

The null and alternative hypotheses. The use of $\sum_{i=1}^n \frac{(O_i - E_i)^2}{E_i}$ as an approximate χ^2 statistic.

Degrees of freedom.

Applications to include the discrete uniform, binomial, Normal, Poisson and continuous uniform (rectangular) distributions. Lengthy calculations will not be required.

Students will be expected to determine the degrees of freedom when one or more parameters are estimated from the data. Cells should be combined when $E_i < 5$. Yates' correction is not required.

5 Regression and correlation

What students need to learn:

Spearman's rank correlation coefficient, its use, interpretation and limitations.

Testing the hypothesis that a correlation is zero.

Numerical questions involving ties will not be set. Some understanding of how to deal with ties will be expected.

Use of tables for Spearman's and product moment correlation coefficients.

S4.1 Unit description

Quality of tests and estimators; one-sample procedures; two-sample procedures.

S4.2 Assessment information**Prerequisites**

A knowledge of the specification for S1, S2 and S3 and its prerequisites and associated formulae is assumed and may be tested.

Examination

The examination will consist of one 1½ hour paper. The paper will contain about seven questions with varying mark allocations per question which will be stated on the paper. All questions may be attempted.

Calculators

Students are expected to have available a calculator with at least the following keys: +, −, ×, ÷, π, x^2 , \sqrt{x} , $\frac{1}{x}$, x^y , $\ln x$, e^x , $x!$, sine, cosine and tangent and their inverses in degrees and decimals of a degree, and in radians; memory. Calculators with a facility for symbolic algebra, differentiation and/or integration are not permitted.

Formulae

Students are expected to know any other formulae which might be required by the specification and which are not included in the booklet, *Mathematical Formulae including Statistical Formulae and Tables*, which will be provided for use with the paper. Questions will be set in SI units and other units in common usage.

Students will be expected to know and be able to recall and use the following formulae:

Type I error: $P(\text{reject } H_0 \mid H_0 \text{ true})$

Type II error: $P(\text{do not reject } H_0 \mid H_0 \text{ false})$

1 Quality of tests and estimators

What students need to learn:

Type I and Type II errors.

Size and Power of Test.

The power test.

Assessment of the quality of estimators.

Simple applications. Calculation of the probability of a Type I or Type II error. Use of Type I and Type II errors and power function to indicate effectiveness of statistical tests. Questions will not be restricted to the Normal distribution.

Use of bias and the variance of an estimator to assess its quality. Consistent estimators.

2 One-sample procedures

What students need to learn:

Hypothesis test and confidence interval for the mean of a Normal distribution with unknown variance.

Hypothesis test and confidence interval for the variance of a Normal distribution.

Use of $\frac{\bar{X} - \mu}{S/\sqrt{n}} \sim t_{n-1}$. Use of t -tables.

Use of $\frac{(n-1)S^2}{\sigma^2} \sim \chi^2_{n-1}$.

Use of χ^2 tables.

3 Two-sample procedures

What students need to learn:

Hypothesis test that two independent random samples are from Normal populations with equal variances.

Use of $\frac{S_1^2}{S_2^2} \sim F_{n_1-1, n_2-1}$ under H_0 .

Use of the tables of the F -distribution.

Use of the pooled estimate of variance.

$$s^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}$$

Hypothesis test and confidence interval for the difference between two means from independent Normal distributions when the variances are equal but unknown.

Use of t -distribution.

$$\frac{\bar{X} - \bar{Y}}{S \sqrt{\frac{1}{n_x} + \frac{1}{n_y}}} \sim t_{n_x + n_y - 2}, \text{ under } H_0.$$

Paired t -test.

Unit D1 Decision Mathematics 1

GCE AS and GCE Mathematics, GCE AS and GCE Further Mathematics and GCE AS and GCE Further Mathematics (Additional) AS optional unit

D1.1 Unit description

Algorithms; algorithms on graphs; the route inspection problem; critical path analysis; linear programming; matchings.

D1.2 Assessment information

Preamble

Students should be familiar with the terms defined in the glossary attached to this specification. Students should show clearly how an algorithm has been applied. Matrix representation will be required but matrix manipulation is not required. Students will be required to model and interpret situations, including cross-checking between models and reality.

Examination

The examination will consist of one 1½ hour paper. The paper will contain about seven questions with varying mark allocations per question which will be stated on the paper. All questions may be attempted.

Calculators

Students are expected to have available a calculator with at least the following keys: +, −, ×, ÷, π, x^2 , \sqrt{x} , $\frac{1}{x}$, x^y and memory. Calculators with a facility for symbolic algebra, differentiation and/or integration are not permitted.

Formulae

Students are expected to know any other formulae which might be required by the specification and which are not included in the booklet, *Mathematical Formulae including Statistical Formulae and Tables*, which will be provided for use with the paper. Questions will be set in SI units and other units in common usage.

1 Algorithms

What students need to learn:

The general ideas of algorithms and the implementation of an algorithm given by a flow chart or text.

The order of an algorithm is not expected.

Whenever finding the middle item of any list, the method defined in the glossary must be used.

Students should be familiar with bin packing, bubble sort, quick sort, binary search.

When using the quick sort algorithm, the pivot should be chosen as the middle item of the list.

2 Algorithms on graphs

What students need to learn:

The minimum spanning tree (minimum connector) problem. Prim's and Kruskal's (greedy) algorithm.

Matrix representation for Prim's algorithm is expected. Drawing a network from a given matrix and writing down the matrix associated with a network will be involved.

Dijkstra's algorithm for finding the shortest path.

3 The route inspection problem

What students need to learn:

Algorithm for finding the shortest route around a network, travelling along every edge at least once and ending at the start vertex. The network will have up to four odd nodes.

Also known as the 'Chinese postman' problem. Students will be expected to use inspection to consider all possible pairings of odd nodes.

(The application of Floyd's algorithm to the odd nodes is not required.)

4 Critical path analysis

What students need to learn:

Modelling of a project by an activity network, from a precedence table.

Activity on arc will be used. The use of dummies is included. In a precedence network, precedence tables will only show immediate predecessors.

Completion of the precedence table for a given activity network.

Algorithm for finding the critical path. Earliest and latest event times. Earliest and latest start and finish times for activities.

Total float. Gantt (cascade) charts. Scheduling.

5 Linear programming

What students need to learn:

Formulation of problems as linear programs.

Graphical solution of two variable problems using ruler and vertex methods

Consideration of problems where solutions must have integer values.

6 Matchings

What students need to learn:

Use of bipartite graphs for modelling matchings. Complete matchings and maximal matchings.

Students will be required to use the maximum matching algorithm to improve a matching by finding alternating paths. No consideration of assignment is required.

Algorithm for obtaining a maximum matching.

Glossary for D1

1 Algorithms

In a list containing N items the 'middle' item has position $\lceil \frac{1}{2}(N+1) \rceil$ if N is odd $\lfloor \frac{1}{2}(N+2) \rfloor$ if N is even, so that if $N=9$, the middle item is the 5th and if $N=6$ it is the 4th.

2 Algorithms on graphs

A **graph** G consists of points (**vertices** or **nodes**) which are connected by lines (**edges** or **arcs**).

A **subgraph** of G is a graph, each of whose vertices belongs to G and each of whose edges belongs to G .

If a graph has a number associated with each edge (usually called its **weight**) then the graph is called a **weighted graph** or **network**.

The **degree** or **valency** of a vertex is the number of edges incident to it. A vertex is **odd (even)** if it has **odd (even)** degree.

A **path** is a finite sequence of edges, such that the end vertex of one edge in the sequence is the start vertex of the next, and in which no vertex appears more than once.

A **cycle (circuit)** is a closed path, ie the end vertex of the last edge is the start vertex of the first edge.

Two vertices are **connected** if there is a path between them. A graph is **connected** if all its vertices are connected.

If the edges of a graph have a direction associated with them they are known as **directed edges** and the graph is known as a **digraph**.

A **tree** is a connected graph with no cycles.

A **spanning tree** of a graph G is a subgraph which includes all the vertices of G and is also a tree.

A **minimum spanning tree (MST)** is a spanning tree such that the total length of its arcs is as small as possible. (MST is sometimes called a **minimum connector**.)

A graph in which each of the n vertices is connected to every other vertex is called a **complete graph**.

4 Critical path analysis

The **total float** $F(i, j)$ of activity (i, j) is defined to be $F(i, j) = l_j - e_i - \text{duration}(i, j)$, where e_i is the earliest time for event i and l_j is the latest time for event j .

6 Matchings

A **bipartite graph** consists of two sets of vertices X and Y . The edges only join vertices in X to vertices in Y , not vertices within a set. (If there are r vertices in X and s vertices in Y then this graph is $K_{r,s}$.)

A **matching** is the pairing of some or all of the elements of one set, X , with elements of a second set, Y . If every member of X is paired with a member of Y the matching is said to be a **complete matching**.

Unit D2 Decision Mathematics 2

GCE Mathematics, GCE AS and GCE Further Mathematics, GCE AS and GCE Further Mathematics (Additional) A2 optional unit

D2.1 Unit description

Transportation problems; allocation (assignment) problems; the travelling salesman; game theory; further linear programming, dynamic programming; flows in networks

D2.2 Assessment information

Prerequisites and Preamble

Prerequisites

Students should be familiar with the material in, and the glossary attached to, the D1 specification.

Preamble

Students should be familiar with the terms defined in the glossary attached to this specification. Students should show clearly how an algorithm has been applied.

Examination

The examination will consist of one 1½ hour paper. The paper will contain about seven questions with varying mark allocations per question which will be stated on the paper. All questions may be attempted.

Calculators

Students are expected to have available a calculator with at least the following keys: +, −, ×, ÷, π, x^2 , \sqrt{x} , $\frac{1}{x}$, x^y and memory. Calculators with a facility for symbolic algebra, differentiation and/or integration are not permitted.

Formulae

Students are expected to know any other formulae which might be required by the specification and which are not included in the booklet, *Mathematical Formulae including Statistical Formulae and Tables*, which will be provided for use with the paper. Questions will be set in SI units and other units in common usage.

1 Transportation problems

What students need to learn:

The north-west corner method for finding an initial basic feasible solution.

Problems will be restricted to a maximum of four sources and four destinations. The idea of dummy locations and degeneracy are required.

Use of the stepping-stone method for obtaining an improved solution. Improvement indices.

Students should identify a specific entering cell and a specific exiting cell.

Formulation as a linear programming problem.

2 Allocation (assignment) problems

What students need to learn:

Cost matrix reduction.

Students should reduce rows first.

The size of the cost matrix will be at most 5×5 .

Use of the Hungarian algorithm to find least cost allocation.

The idea of a dummy location is required.

Modification of method to deal with a maximum profit allocation.

Formulation as a linear programming problem.

3 The travelling salesman problem

What students need to learn:

The practical and classical problems. The classical problem for complete graphs satisfying the triangle inequality.

Determination of upper and lower bounds using minimum spanning tree methods.

The nearest neighbour algorithm.

The use of short cuts to improve upper bound is included.

The conversion of a network into a complete network of shortest 'distances' is included.

4 Further linear programming

What students need to learn:

Formulation of problems as linear programs.

The Simplex algorithm and tableau for maximising problems.

The use and meaning of slack variables.

Problems will be restricted to those with a maximum of four variables and four constraints, in addition to non-negativity conditions.

5 Game theory

What students need to learn:

Two person zero-sum games and the pay-off matrix. Identification of play safe strategies and stable solutions (saddle points).

Reduction of pay-off matrices using dominance arguments. Optimal mixed strategies for a game with no stable solution.

Conversion of 3×2 and 3×3 games to linear programming problems.

Use of graphical methods for $2 \times n$ or $n \times 2$ games where $n = 1, 2$ or 3 .

6 Flows in networks

What students need to learn:

Algorithm for finding a maximum flow.

Cuts and their capacity.

Use of the max flow — min cut theorem to verify that a flow is a maximum flow.

Multiple sources and sinks.

Vertex restrictions are not required. Only networks with directed arcs will be considered. Only problems with upper capacities will be set.

7 Dynamic programming

What students need to learn:

Principles of dynamic programming. Bellman's principle of optimality.

Stage variables and State variables. Use of tabulation to solve maximum, minimum, minimax or maximin problems.

Both network and table formats are required.

Glossary for D2

1 Transportation problems

In the **north-west corner method**, the upper left-hand cell is considered first and as many units as possible sent by this route.

The **stepping stone method** is an iterative procedure for moving from an initial feasible solution to an optimal solution.

Degeneracy occurs in a transportation problem, with m rows and n columns, when the number of occupied cells is less than $(m + n - 1)$.

In the transportation problem:

The **shadow costs** R_i , for the i th row, and K_j , for the j th column, are obtained by solving $R_i + K_j = C_{ij}$ for **occupied cells**, taking $R_1 = 0$ arbitrarily.

The **improvement index** I_{ij} for an **unoccupied cell** is defined by $I_{ij} = C_{ij} - R_i - K_j$.

3 The travelling salesman problem

The **travelling salesman problem** is ‘find a route of minimum length which visits every vertex in an undirected network’. In the ‘**classical**’ problem, each vertex is visited once only. In the ‘**practical**’ problem, a vertex may be revisited.

For three vertices A , B and C , the **triangular inequality** is ‘length $AB \leq$ length $AC +$ length CB ’.

A **walk** in a network is a finite sequence of edges such that the end vertex of one edge is the start vertex of the next.

A walk which visits every vertex, returning to its starting vertex, is called a **tour**.

4 Further linear programming

The **simplex tableau** for the linear programming problem:

$$\begin{array}{ll} \text{Maximise} & P = 14x + 12y + 13z, \\ \text{Subject to} & 4x + 5y + 3z \leq 16, \\ & 5x + 4y + 6z \leq 24, \end{array}$$

will be written as

Basic variable	x	y	z	r	s	Value
r	4	5	3	1	0	16
s	5	4	6	0	1	24
P	-14	-12	-13	0	0	0

where r and s are slack variables.

5 Game theory

A **two-person game** is one in which only two parties can play.

A **zero-sum game** is one in which the sum of the losses for one player is equal to the sum of the gains for the other player.

6 Flows in networks

A **cut**, in a network with source S and sink T , is a set of arcs (edges) whose removal separates the network into two parts X and Y , where X contains at least S and Y contains at least T . The **capacity of a cut** is the sum of the capacities of those arcs in the cut which are directed from X to Y .

If a network has several sources S_1, S_2, \dots , then these can be connected to a single **supersource** S . The capacity of the edge joining S to S_1 is the sum of the capacities of the edges leaving S_1 .

If a network has several sinks T_1, T_2, \dots , then these can be connected to a **supersink** T . The capacity of the edge joining T_1 to T is the sum of the capacities of the edges entering T_1 .

7 Dynamic programming

Bellman's principle for dynamic programming is 'Any part of an optimal path is optimal.'

The **minimax route** is the one in which the maximum length of the arcs used is as small as possible.

The **maximin route** is the one in which the minimum length of the arcs used is as large as possible.

D Assessment and additional information

Assessment information

Assessment requirements

For a summary of assessment requirements and assessment objectives, see *Section B, Specification overview*.

Entering students for the examinations for this qualification

Details of how to enter students for the examinations for this qualification can be found in Edexcel's *Information Manual*, copies of which are sent to all examinations officers. The information can also be found on Edexcel's website (www.edexcel.org.uk).

Resitting of units

There is no restriction on the number of times a unit can be attempted prior to claiming certification for the qualification. The best available result for each contributing unit will count towards the final grade.

Results of units will be held in Edexcel's unit bank for as many years as this specification remains available. Once the Advanced Subsidiary (AS) or Advanced Level qualification has been certificated, all unit results are deemed to be used up at that level. These results cannot be used again towards a further award of the same qualification at the same level but unit results used for an AS award remain available for use in an Advanced GCE level award.

Awarding and reporting

The grading, awarding and certification of this qualification will comply with the requirements of the current GCSE/GCE Code of Practice for courses starting in September 2008, which is published by the Qualifications and Curriculum Authority. The AS qualification will be graded and certificated on a five-grade scale from A to E. The full GCE Advanced level will be graded on a six-point scale A* to E. Individual unit results will be reported.

A pass in an Advanced Subsidiary subject is indicated by one of the five grades A, B, C, D, E of which grade A is the highest and grade E the lowest. A pass in an Advanced GCE subject is indicated by one of the six grades A*, A, B, C, D, E of which Grade A* is the highest and Grade E the lowest. To be awarded an A* students will need to achieve an A on the full GCE Advanced level qualification and an A* aggregate of the A2 units. Students whose level of achievement is below the minimum judged by Edexcel to be of sufficient standard to be recorded on a certificate will receive an unclassified U result.

Performance descriptions

Performance descriptions give the minimum acceptable level for a grade. See *Appendix A* for the performance descriptions for this subject.

Unit results

The minimum uniform marks required for each grade for each unit:

Unit grade	A	B	C	D	E
Maximum uniform mark = 100	80	70	60	50	40

Students who do not achieve the standard required for a Grade E will receive a uniform mark in the range 0–39.

Qualification results

The minimum uniform marks required for each grade:

Advanced Subsidiary Cash in code 8371, 8372, 8373, 8374

Qualification grade	A	B	C	D	E
Maximum uniform mark = 300	240	210	180	150	120

Students who do not achieve the standard required for a Grade E will receive a uniform mark in the range 0–119.

Advanced GCE Cash in code 9371, 9372, 9373, 9374

Qualification grade	A	B	C	D	E
Maximum uniform mark = 600	480	420	360	300	240

Students who do not achieve the standard required for a Grade E will receive a uniform mark in the range 0–239.

Language of assessment

Assessment of this specification will be available in English only. Assessment materials will be published in English only and all work submitted for examination and moderation must be produced in English.

Quality of written communication

Students will be assessed on their ability to:

- write legibly, with accurate use of spelling, grammar and punctuation in order to make the meaning clear
- select and use a form and style of writing appropriate to purpose and to complex subject matter
- organise relevant information clearly and coherently, using specialist vocabulary when appropriate.

Assessment objectives and weighting

	The assessment will test students' ability to:	Minimum weighting
AO1	recall, select and use their knowledge of mathematical facts, concepts and techniques in a variety of contexts	30%
AO2	construct rigorous mathematical arguments and proofs through use of precise statements, logical deduction and inference and by the manipulation of mathematical expressions, including the construction of extended arguments for handling substantial problems presented in unstructured form	30%
AO3	recall, select and use their knowledge of standard mathematical models to represent situations in the real world; recognise and understand given representations involving standard models; present and interpret results from such models in terms of the original situation, including discussion of the assumptions made and refinement of such models	10%
AO4	comprehend translations of common realistic contexts into mathematics; use the results of calculations to make predictions, or comment on the context; and, where appropriate, read critically and comprehend longer mathematical arguments or examples of applications	5%
AO5	use contemporary calculator technology and other permitted resources (such as formulae booklets or statistical tables) accurately and efficiently; understand when not to use such technology, and its limitations. Give answers to appropriate accuracy.	5%

Synoptic assessment

In synoptic assessment there should be a concentration on the quality of assessment to ensure that it encourages the development of the holistic understanding of the subject.

Synopticity requires students to connect knowledge, understanding and skills acquired in different parts of the Advanced GCE course.

Synoptic assessment in mathematics addresses students' understanding of the connections between different elements of the subject. It involves the explicit drawing together of knowledge, understanding and skills learned in different parts of the Advanced GCE course through using and applying methods developed at earlier stages of study in solving problems. Making and understanding connections in this way is intrinsic to learning mathematics.

In papers which address the A2 core content, synoptic assessment requires the use of methods from the AS core content. In papers which address mathematical content outside the core content, synoptic assessment requires the use of methods from the core content and/or methods from earlier stages of the same aspect of mathematics (pure mathematics, mechanics, statistics or decision mathematics).

Synoptic assessment is addressed in the assessment objectives as parts of AO1, AO2, AO3, AO4 and AO5 and represents 20% of the assessment for Advanced GCE.

Additional information**Malpractice and plagiarism**

For up-to-date advice on malpractice and plagiarism, please refer to the Joint Council for Qualifications (JCQ) Suspected Malpractice in Examinations: Policies and Procedures document on the JCQ website (www.jcq.org.uk).

Access arrangements and special requirements

Edexcel's policy on access arrangements and special considerations for GCE, GCSE, and Entry Level aims to enhance access to the qualifications for learners with disabilities and other difficulties (as defined by the Disability Discrimination Act 1995 and the amendments to the Act) without compromising the assessment of skills, knowledge, understanding or competence.

Please see the Edexcel website (www.edexcel.org.uk/sfc) for:

- the JCQ policy *Access Arrangements and Special Considerations, Regulations and Guidance Relating to Students who are Eligible for Adjustments in Examinations*
- the forms to submit for requests for access arrangements and special considerations
- dates for submission of the forms.

Requests for access arrangements and special considerations must be addressed to:

Special Requirements
Edexcel
One90 High Holborn
London WC1V 7BH

Disability Discrimination Act

Please see *Appendix G* for Advanced GCE in Mathematics Disability Discrimination Act information.

Prior learning and progression

Prior learning

Students embarking on Advanced Subsidiary and Advanced GCE study in Mathematics are expected to have covered all the material in the GCSE Mathematics Higher Tier. This material is regarded as assumed background knowledge and will not be tested by questions focused directly on it. However, it may be assessed within questions focused on other material from the relevant specification.

Progression

This qualification supports progression into further and higher education, training or employment, in a wide variety of disciplines.

Combinations of entry

Any two subjects which have an overlap of at least one unit form a forbidden combination. Any two subjects with the same title also form a forbidden combination, whether or not there is any overlap of units.

Every specification is assigned to a national classification code indicating the subject area to which it belongs.

Centres should be aware that students who enter for more than one GCE qualification with the same classification code, will have only one grade (the highest) counted for the purpose of the School and College Performance Tables.

Please see *Appendix E* for further details of the classification codes.

Conditions of dependency

The units in the areas of Pure Mathematics, Mechanics, Statistics and Decision Mathematics are consecutively numbered in order of study. The study of a unit is dependent on the study of all preceding units within that area of mathematics; for example, the study of C3 is dependent on the study of C1 and C2.

Students who wish to take Advanced Subsidiary or Advanced GCE in Further Mathematics may be expected to have obtained (or to be obtaining concurrently) an Advanced Subsidiary or Advanced GCE in Mathematics. Units that contribute to an award in A level Mathematics may not also be used for an award in Further Mathematics. Students who have obtained or who are in the process of obtaining Advanced GCE in Mathematics with an awarding body other than Edexcel should contact the awarding body to check requirements for Further Mathematics.

Student recruitment

Edexcel's access policy concerning recruitment to our qualifications is that:

- they must be available to anyone who is capable of reaching the required standard
- they must be free from barriers that restrict access and progression
- equal opportunities exist for all students.

Key skills

This qualification provides opportunities for developing and generating evidence for assessing the key skills listed below:

- application of number
- communication
- information and communication technology
- improving own learning and performance
- problem solving
- working with others.

Full details are available in *Appendices C and D*.

This qualification will be mapped to functional skills once they are finalised. Information will be available on our website (www.edexcel.org.uk/gce2008) at a later date.

The wider curriculum

This qualification provides opportunities for developing an understanding of spiritual, moral, ethical, social and cultural issues, together with an awareness of environmental issues, health and safety considerations, and European developments consistent with relevant international agreements. *Appendix C: Key Skills Mapping* maps the opportunities available.

Resources to support the specification

In addition to the resources available in the e-Spec and in the *Getting Started* guide book, Edexcel produces a wide range of resources to support this specification.

Edexcel's own published resources

Edexcel aims to provide the most comprehensive support for our qualifications. We have therefore published our own dedicated suite of resources for teachers and students written by qualification experts.

The resources for Mathematics (Edexcel) include:

- AS Students' Book
- A2 Students' Book
- AS ActiveTeach CD ROM
- A2 ActiveTeach CD ROM
- AS Teacher Support Pack
- A2 Teacher Support Pack.

For more information on our complete range of products and services for GCE in Mathematics, visit www.edexcel.org.uk/gce2008.

Edexcel publications

You can order further copies of the specification and sample assessment materials (SAMs) documents from:

Edexcel Publications
Adamsway
Mansfield
Nottinghamshire
NG18 4FN

Telephone: 01623 467467
Fax: 01623 450481
Email: publications@linneydirect.com
Website: www.edexcel.org.uk

Additional resources endorsed by Edexcel

Edexcel also endorses additional materials written to support this qualification.

Any resources bearing the 'Endorsed by Edexcel' logo have been through a rigorous quality assurance process to ensure complete and accurate support for the specification. For up-to-date information about endorsed resources, please visit www.edexcel.org.uk/endorsed

Please note that while resources are checked at the time of publication, materials may be withdrawn from circulation and website locations may change.

Please see www.edexcel.org.uk/gce2008 for up-to-date information.

Support

Edexcel support services

Edexcel has a wide range of support services to help you implement this qualification successfully.

ResultsPlus — ResultsPlus is a new application launched by Edexcel to help subject teachers, senior management teams, and students by providing detailed analysis of examination performance. Reports that compare performance between subjects, classes, your centre and similar centres can be generated in 'one-click'. Skills maps that show performance according to the specification topic being tested are available for some subjects. For further information about which subjects will be analysed through ResultsPlus, and for information on how to access and use the service, please visit www.edexcel.org.uk/resultsplus.

Ask the Expert — Ask the Expert is a new service, launched in 2007, that provides direct email access to senior subject specialists who will be able to answer any questions you might have about this or any other specification. All of our specialists are senior examiners, moderators or verifiers and they will answer your email personally. You can read a biography for all of them and learn more about this unique service on our website at www.edexcel.org.uk/asktheexpert.

Ask Edexcel — Ask Edexcel is Edexcel’s online question and answer service. You can access it at www.edexcel.org.uk/ask or by going to the main website and selecting the Ask Edexcel menu item on the left.

The service allows you to search through a database of thousands of questions and answers on everything Edexcel offers. If you don’t find an answer to your question, you can choose to submit it straight to us. One of our customer services team will log your query, find an answer and send it to you. They’ll also consider adding it to the database if appropriate. This way the volume of helpful information that can be accessed via the service is growing all the time.

Examzone — The Examzone site is aimed at students sitting external examinations and gives information on revision, advice from examiners and guidance on results, including re-marking, re-sitting and progression opportunities. Further services for students — many of which will also be of interest to parents — will be available in the near future. Links to this site can be found on the main homepage at www.edexcel.org.uk.

Training

A programme of professional development and training courses, covering various aspects of the specification and examination, will be arranged by Edexcel each year on a regional basis. Full details can be obtained from:

Professional Development and Training
Edexcel
One90 High Holborn
London WC1V 7BH

Telephone: 0870 240 9800
Fax: 0845 359 1909
Email: trainingenquiries@edexcel.org.uk
Website: www.edexcel.org.uk

F Appendices

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The following grade descriptions indicate the level of attainment characteristic of grades A, C and E at Advanced GCE. They give a general indication of the required learning outcomes at the specified grades. The descriptions should be interpreted in relation to the content outlined in the specification; they are not designed to define that content. The grade awarded will depend in practice upon the extent to which the candidate has met the assessment objectives overall. Shortcomings in some aspects of the examination may be balanced by better performances in others.

Grade A

Students recall or recognise almost all the mathematical facts, concepts and techniques that are needed, and select appropriate ones to use in a wide variety of contexts.

Students manipulate mathematical expressions and use graphs, sketches and diagrams, all with high accuracy and skill. They use mathematical language correctly and proceed logically and rigorously through extended arguments or proofs. When confronted with unstructured problems they can often devise and implement an effective solution strategy. If errors are made in their calculations or logic, these are sometimes noticed and corrected.

Students recall or recognise almost all the standard models that are needed, and select appropriate ones to represent a wide variety of situations in the real world. They correctly refer results from calculations using the model to the original situation; they give sensible interpretations of their results in the context of the original realistic situation. They make intelligent comments on the modelling assumptions and possible refinements to the model.

Students comprehend or understand the meaning of almost all translations into mathematics of common realistic contexts. They correctly refer the results of calculations back to the given context and usually make sensible comments or predictions. They can distil the essential mathematical information from extended pieces of prose having mathematical content. They can comment meaningfully on the mathematical information.

Students make appropriate and efficient use of contemporary calculator technology and other permitted resources, and are aware of any limitations to their use. They present results to an appropriate degree of accuracy.

Grade C

Students recall or recognise most of the mathematical facts, concepts and techniques that are needed, and usually select appropriate ones to use in a variety of contexts.

Students manipulate mathematical expressions and use graphs, sketches and diagrams, all with a reasonable level of accuracy and skill. They use mathematical language with some skill and sometimes proceed logically through extended arguments or proofs. When confronted with unstructured problems they sometimes devise and implement an effective and efficient solution strategy. They occasionally notice and correct errors in their calculations.

Students recall or recognise most of the standard models that are needed and usually select appropriate ones to represent a variety of situations in the real world. They often correctly refer results from calculations using the model to the original situation; they sometimes give sensible interpretations of their results in the context of the original realistic situation. They sometimes make intelligent comments on the modelling assumptions and possible refinements to the model.

Students comprehend or understand the meaning of most translations into mathematics of common realistic contexts. They often correctly refer the results of calculations back to the given context and sometimes make sensible comments or predictions. They distil much of the essential mathematical information from extended pieces of prose having mathematical content. They give some useful comments on this mathematical information.

Students usually make appropriate and efficient use of contemporary calculator technology and other permitted resources, and are sometimes aware of any limitations to their use. They usually present results to an appropriate degree of accuracy.

Grade E

Students recall or recognise some of the mathematical facts, concepts and techniques that are needed, and sometimes select appropriate ones to use in some contexts.

Students manipulate mathematical expressions and use graphs, sketches and diagrams, all with some accuracy and skill. They sometimes use mathematical language correctly and occasionally proceed logically through extended arguments or proofs.

Students recall or recognise some of the standard models that are needed and sometimes select appropriate ones to represent a variety of situations in the real world. They sometimes correctly refer results from calculations using the model to the original situation; they try to interpret their results in the context of the original realistic situation.

Students sometimes comprehend or understand the meaning of translations in mathematics of common realistic contexts. They sometimes correctly refer the results of calculations back to the given context and attempt to give comments or predictions. They distil some of the essential mathematical information from extended pieces of prose having mathematical content. They attempt to comment on this mathematical information.

Students often make appropriate and efficient use of contemporary calculator technology and other permitted resources. They often present results to an appropriate degree of accuracy.

Appendix B Notation

The following notation will be used in all mathematics examinations:

1 Set notation

\in	is an element of
\notin	is not an element of
$\{x_1, x_2, \dots\}$	the set with elements x_1, x_2, \dots
$\{x:\dots\}$	the set of all x such that ...
$n(A)$	the number of elements in set A
\emptyset	the empty set
\mathcal{E}	the universal set
A'	the complement of the set A
\mathbb{N}	the set of natural numbers, $\{1, 2, 3, \dots\}$
\mathbb{Z}	the set of integers, $\{0, \pm 1, \pm 2, \pm 3, \dots\}$
\mathbb{Z}^+	the set of positive integers, $\{1, 2, 3, \dots\}$
\mathbb{Z}_n	the set of integers modulo n , $\{0, 1, 2, \dots, n-1\}$
\mathbb{Q}	the set of rational numbers, $\left\{\frac{p}{q} : p \in \mathbb{Z}, q \in \mathbb{Z}^+\right\}$
\mathbb{Q}^+	the set of positive rational numbers, $\{x \in \mathbb{Q} : x > 0\}$
\mathbb{Q}_0^+	the set of positive rational numbers and zero, $\{x \in \mathbb{Q} : x \geq 0\}$
\mathbb{R}	the set of real numbers
\mathbb{R}^+	the set of positive real numbers $\{x \in \mathbb{R} : x > 0\}$
\mathbb{R}_0^+	the set of positive real numbers and zero, $\{x \in \mathbb{R} : x \geq 0\}$
\mathbb{C}	the set of complex numbers
(x, y)	the ordered pair x, y
$A \times B$	the cartesian product of sets A and B , ie $A \times B = \{(a, b) : a \in A, b \in B\}$
\subseteq	is a subset of
\subset	is a proper subset of
\cup	union
\cap	intersection

$[a, b]$	the closed interval, $\{x \in \mathbb{R} : a \leq x \leq b\}$
$[a, b), [a, b[$	the interval $\{x \in \mathbb{R} : a \leq x < b\}$
$(a, b],]a, b]$	the interval $\{x \in \mathbb{R} : a < x \leq b\}$
$(a, b),]a, b[$	the open interval $\{x \in \mathbb{R} : a < x < b\}$
$y R x$	y is related to x by the relation R
$y \sim x$	y is equivalent to x , in the context of some equivalence relation

2 Miscellaneous symbols

$=$	is equal to
\neq	is not equal to
\equiv	is identical to or is congruent to
\approx	is approximately equal to
\cong	is isomorphic to
\propto	is proportional to
$<$	is less than
\leq	is less than or equal to, is not greater than
$>$	is greater than
\geq	is greater than or equal to, is not less than
∞	infinity
$p \wedge q$	p and q
$p \vee q$	p or q (or both)
$\sim p$	not p
$p \Rightarrow q$	p implies q (if p then q)
$p \Leftarrow q$	p is implied by q (if q then p)
$p \Leftrightarrow q$	p implies and is implied by q (p is equivalent to q)
\exists	there exists
\forall	for all

3 Operations

$a + b$

a plus b

$a - b$

a minus b

$a \times b, ab, a.b$

a multiplied by b

$a \div b, \frac{a}{b}, a/b$

a divided by b

$$\sum_{i=1}^n a_i$$

$$a_1 + a_2 + \dots + a_n$$

$$\prod_{i=1}^n a_i$$

$$a_1 \times a_2 \times \dots \times a_n$$

\sqrt{a}

the positive square root of a

$|a|$

the modulus of a

$n!$

n factorial

$$\binom{n}{r}$$

the binomial coefficient $\frac{n!}{r! (n-r)!}$ for $n \in \mathbb{Z}^+$
 $\frac{n(n-1) \dots (n-r+1)}{r!}$ for $n \in \mathbb{Q}$

4 Functions

$f(x)$	the value of the function f at x
$f : A \rightarrow B$	f is a function under which each element of set A has an image in set B
$f : x \rightarrow y$	the function f maps the element x to the element y
f^{-1}	the inverse function of the function f
$g \circ f, gf$	the composite function of f and g which is defined by $(g \circ f)(x)$ or $gf(x) = g(f(x))$
$\lim_{x \rightarrow a} f(x)$	the limit of $f(x)$ as x tends to a
$\Delta x, \delta x$	an increment of x
$\frac{dy}{dx}$	the derivative of y with respect to x
$\frac{d^n y}{dx^n}$	the n th derivative of y with respect to x
$f'(x), f''(x), \dots, f^{(n)}(x)$	the first, second, ... , n th derivatives of $f(x)$ with respect to x
$\int y \, dx$	the indefinite integral of y with respect to x
$\int_a^b y \, dx$	the definite integral of y with respect to x between the limits $x = a$ and $x = b$
$\frac{\partial V}{\partial x}$	the partial derivative of V with respect to x
\dot{x}, \ddot{x}, \dots	the first, second, ... derivatives of x with respect to t

5 Exponential and logarithmic functions

e	base of natural logarithms
$e^x, \exp x$	exponential function of x
$\log_a x$	logarithm to the base a of x
$\ln x, \log_e x$	natural logarithm of x
$\lg x, \log_{10} x$	logarithm of x to base 10

6 Circular and hyperbolic functions

$\left. \begin{array}{l} \sin, \cos, \tan, \\ \operatorname{cosec}, \sec, \cot \end{array} \right\}$	the circular functions
$\left. \begin{array}{l} \arcsin, \arccos, \arctan, \\ \operatorname{arccosec}, \operatorname{arcsec}, \operatorname{arccot} \end{array} \right\}$	the inverse circular functions
$\left. \begin{array}{l} \sinh, \cosh, \tanh, \\ \operatorname{cosech}, \operatorname{sech}, \operatorname{coth} \end{array} \right\}$	the hyperbolic functions
$\left. \begin{array}{l} \operatorname{arsinh}, \operatorname{arcosh}, \operatorname{artanh}, \\ \operatorname{arcosech}, \operatorname{arsech}, \operatorname{arcoth} \end{array} \right\}$	the inverse hyperbolic functions

7 Complex numbers

i, j	square root of -1
z	a complex number, $z = x + iy$
$\operatorname{Re} z$	the real part of z , $\operatorname{Re} z = x$
$\operatorname{Im} z$	the imaginary part of z , $\operatorname{Im} z = y$
$ z $	the modulus of z , $ z = \sqrt{(x^2 + y^2)}$
$\arg z$	the argument of z , $\arg z = \theta$, $-\pi < \theta \leq \pi$
z^*	the complex conjugate of z , $x - iy$

8 Matrices

\mathbf{M}	a matrix \mathbf{M}
\mathbf{M}^{-1}	the inverse of the matrix \mathbf{M}
\mathbf{M}^T	the transpose of the matrix \mathbf{M}
$\det \mathbf{M}$ or $ \mathbf{M} $	the determinant of the square matrix

9 Vectors

\mathbf{a}	the vector \mathbf{a}
\vec{AB}	the vector represented in magnitude and direction by the directed line segment AB
$\hat{\mathbf{a}}$	a unit vector in the direction of \mathbf{a}
$\mathbf{i}, \mathbf{j}, \mathbf{k}$	unit vectors in the directions of the cartesian coordinate axes
$ \mathbf{a} , a$	the magnitude of \mathbf{a}
$ \vec{AB} , AB$	the magnitude of \vec{AB}
$\mathbf{a} \cdot \mathbf{b}$	the scalar product of \mathbf{a} and \mathbf{b}
$\mathbf{a} \times \mathbf{b}$	the vector product of \mathbf{a} and \mathbf{b}

10 Probability and statistics

A, B, C , etc	events
$A \cup B$	union of the events A and B
$A \cap B$	intersection of the events A and B
$P(A)$	probability of the event A
A'	complement of the event A
$P(A \mid B)$	probability of the event A conditional on the event B
X, Y, R , etc	random variables
x, y, r , etc	values of the random variables X, Y, R , etc
x_1, x_2, \dots	observations
f_1, f_2, \dots	frequencies with which the observations x_1, x_2, \dots occur
$p(x)$	probability function $P(X = x)$ of the discrete random variable X
p_1, p_2	probabilities of the values x_1, x_2, \dots of the discrete random variable X
$f(x), g(x)$	the value of the probability density function of a continuous random variable X
$F(x), G(x)$	the value of the (cumulative) distribution function $P(X \leq x)$ of a continuous random variable X
$E(X)$	expectation of the random variable X
$E[g(X)]$	expectation of $g(X)$
$\text{Var}(X)$	variance of the random variable X
$G(t)$	probability generating function for a random variable which takes the values $0, 1, 2, \dots$
$B(n, p)$	binomial distribution with parameters n and p
$N(\mu, \sigma^2)$	normal distribution with mean μ and variance σ^2
μ	population mean
σ^2	population variance
σ	population standard deviation
\bar{x}, m	sample mean
$s^2, \hat{\sigma}^2$	unbiased estimate of population variance from a sample, $s^2 = \frac{1}{n-1} \sum (x_i - \bar{x})^2$
ϕ	probability density function of the standardised normal variable with distribution $N(0, 1)$
Φ	corresponding cumulative distribution function

ρ	product moment correlation coefficient for a population
r	product moment correlation coefficient for a sample
$\text{Cov}(X, Y)$	covariance of X and Y

Appendix C Key skills mapping

Core and Further Pure Mathematics

Key skills (Level 3)	C1	C2	C3	C4	FP1	FP2	FP3
Application of number							
N3.1	X	X	X				
N3.2	X	X	X				
N3.3	X	X	X				
Communication							
C3.1a							
C3.1b							
C3.2							
C3.3							
Information and communication technology							
ICT3.1							
ICT3.2							
ICT3.3							
Improving own learning and performance							
LP3.1							
LP3.2							
LP3.3							
Problem solving							
PS3.1	X	X	X		X	X	X
PS3.2	X	X	X		X	X	X
PS3.3	X	X	X		X	X	X
Working with others							
WO3.1							
WO3.2							
WO3.3							

Mechanics, Statistics and Decision Mathematics

Key skills (Level 3)	M1	M2	M3	M4	M5	S1	S2	S3	S4	D1	D2
Application of number											
N3.1	X	X	X			X	X	X	X		
N3.2	X	X	X			X	X	X	X		
N3.3	X	X	X			X	X	X	X		
Communication											
C3.1a						X	X	X	X		
C3.1b						X	X	X	X		
C3.2								X			
C3.3								X			
Information and communication technology											
ICT3.1						X	X	X	X		
ICT3.2						X	X	X	X		
ICT3.3						X	X	X	X		
Improving own learning and performance											
LP3.1	X	X	X	X	X	X	X	X	X		
LP3.2	X	X	X	X	X	X	X	X	X		
LP3.3	X	X	X	X	X	X	X	X	X		
Problem solving											
PS3.1	X	X	X	X	X	X	X	X	X		
PS3.2	X	X	X	X	X	X	X	X	X		
PS3.3	X	X	X	X	X	X	X	X	X		
Working with others											
WO3.1						X	X	X	X		
WO3.2						X	X	X	X		
WO3.3						X	X	X	X		

Achievement of key skills is not a requirement of this qualification but it is encouraged. Suggestions of opportunities for the generation of Level 3 key skill evidence are given here.

Application of number — Level 3

Plan and carry out one or more activities that each includes tasks for all three of N3.1, N3.2 (a or b or c or d) and N3.3.

Overall, through one or more activities you must:

- use two different types of sources, including a large data set, ie over 50 items of data (N3.1)
- carry out calculations to do with a, b, c and d (N3.2)
- present findings in two different ways using charts, graphs or diagrams (N3.3).

Key skills portfolio evidence requirement		Opportunities for development or internal assessment
<p>N3.1</p>	<p>Plan an activity and get relevant information from relevant sources.</p>	<p>Students are required to:</p> <ul style="list-style-type: none"> ■ plan how to obtain and use the information required to meet the purpose of their activity ■ obtain the relevant information; and ■ choose appropriate methods for obtaining the results they need and justify their choice. <p>For example, the criteria for N3.1 are partially satisfied when:</p> <ul style="list-style-type: none"> ■ choosing an appropriate degree of accuracy to work to in calculations ■ using indices ■ using compound measures in calculus ■ setting up a set of logical sequences whilst working towards a proof ■ using large and small numbers in exponential growth and decay ■ setting up appropriate differential equations to model a real-life situation ■ set up a mathematical model of a practical situation. <p>NB: Students should be given topics where they have to choose the method of calculation.</p> <p>In completing the Core and Further Pure Mathematics units students would not necessarily:</p> <ul style="list-style-type: none"> ■ plan a substantial and complex activity by breaking it down into a series of tasks ■ obtain relevant information from different sources including a large data set (over 50 items) ■ make accurate and reliable observations over time and use suitable equipment to measure in a variety of appropriate units.

Key skills portfolio evidence requirement		Opportunities for development or internal assessment
<p>N3.2</p> <p>Use this information to carry out multi-stage calculations to do with:</p> <ul style="list-style-type: none"> a amounts or sizes b scales or proportion c handling statistics d using formulae. 	<p>Students must:</p> <ul style="list-style-type: none"> ■ carry out their calculations to appropriate levels of accuracy, clearly showing their methods; and ■ check methods and results to help ensure errors are found and corrected. <p>For example, the criteria for N3.2 are partially satisfied when:</p> <ul style="list-style-type: none"> ■ students perform multi-stage calculations throughout the units, particularly when using calculus in real-life contexts eg functions, integration by substitution and by parts, product, quotient and chain rule, differential equations, etc ■ they choose an appropriate level of accuracy to work to ■ they calculate and use angles when using trigonometry ■ they perform calculations involving rate of change, particularly in practical contexts ■ they form, rearrange and use formulae, equations and expressions ■ they check their work using alternative methods ■ use vectors to calculate lengths and angles in practical situations ■ perform the appropriate calculations in a mathematical model of a real-life situation. <p>In completing the Core and Further Pure Mathematics units students would not necessarily:</p> <ul style="list-style-type: none"> ■ work with a large data set (over 50 items), using measures of average and range to compare distributions and estimate mean, median and range of grouped data. 	

Key skills portfolio evidence requirement		Opportunities for development or internal assessment
<p>N3.3</p>	<p>Interpret the results of your calculations, present your findings and justify your methods.</p>	<p>On the basis of their findings, students must:</p> <ul style="list-style-type: none"> ■ select appropriate methods of presentation and justify their choice ■ present their findings effectively; and ■ explain how the results of their calculations relate to the purpose of their activity. <p>For example, the criteria for N3.3 are partially satisfied when:</p> <ul style="list-style-type: none"> ■ methods of calculation are discussed and justified. Students would benefit from having a choice of possible methods, eg numerical against calculus methods, and being asked to choose and justify their choice ■ efforts are made to consider the models of real-life situations used, with the limitations of the model realised and discussed. Constraints on methods and calculations used should be applied, eg numerically estimating the area under a curve as opposed to calculating through integration ■ using calculus methods in practical situations to arrive at an optimum choice with justification ■ students discuss the benefits of a mathematical technique, eg using vector notation and scalar products to calculate angles in a 3D practical context. <p>In completing the Core and Further Pure Mathematics units students would not necessarily:</p> <ul style="list-style-type: none"> ■ select and use appropriate methods to illustrate findings, show trends and make comparisons ■ draw appropriate conclusions based on their findings, including how possible sources of error might have affected their results.

Evidence

Candidate evidence for application of number could include:

- a description of the substantial and complex activity and tasks. A plan for obtaining and using the information required
- copies of source material, including a note of the large data set and, if applicable, a statement from someone who has checked the accuracy of any measurements or observations
- records of the information they obtained. A justification of methods selected for achieving the required results
- records of the calculations showing methods used and levels of accuracy
- notes of the large data set used and how they checked methods and results
- report of their findings, including justification of their presentation methods and explanations of how their results relate to their activity. At least one chart, one graph and one diagram.

Communication — Level 3

For the Communication Key Skill, students are required to hold discussions and give presentations, read and synthesise information and write documents. Students will be able to develop all of these skills through an appropriate teaching and learning programme based on this Advanced GCE.

The Statistics units offer the most obvious opportunities for gathering evidence and developing this key skill.

Key skills portfolio evidence requirement		Opportunities for development or internal assessment
C3.1a	Take part in a group discussion.	<p>Many of the topics in this specification are suitable as the basis of a group discussion. The discussion must be about a complex subject. This may be based on a number of ideas, some of which may be abstract, very detailed and/or sensitive. Specialist vocabulary may be used in the discussion. During the discussion students should make clear and relevant contributions, and develop points and ideas whilst listening and responding sensitively to others. They should also create opportunities for others to contribute as appropriate.</p> <p>Many topics in the statistics units lend themselves to group discussion, eg the validity of one method of significance testing or sampling against.</p>
C3.1b	Make a formal presentation of at least eight minutes using an image or other support material.	<p>Following a period of research students could be given the opportunity to present their findings to the rest of the group.</p> <p>During the presentation students should speak clearly and use a style that is appropriate to their audience and the subject. The presentation should have a logical structure that allows the audience to follow the sequence of information and ideas. The presentation should include an appropriate range of techniques such as:</p> <ul style="list-style-type: none"> ■ the use of examples to illustrate complex points, audience experience used to involve the audience, tone of voice varied, etc. <p>Where appropriate, images should be used both to illustrate points and to help engage the audience. Images could include charts and diagrams or other statistical diagrams. At least one image should be used to illustrate and help convey a complex point.</p> <p>Students could also make a presentation relating to topics in the specification, eg forming hypotheses and demonstrating how and why they accepted or rejected them.</p>

Key skills portfolio evidence requirement		Opportunities for development or internal assessment
C3.2	<p>Read and synthesise information from at least two documents about the same subject.</p> <p>Each document must be a minimum of 1000 words long.</p>	<p>Students will have a number of opportunities to read and synthesise information from two extended documents. For example, as part of their preparation for the discussion and presentation of a complex subject, students will need to carry out preliminary research. Also, as students undertake research for their coursework they will need to refer to and synthesise information from a variety of sources.</p> <p>Extended documents may include textbooks and reports and articles of more than three pages. At least one of these documents should contain an image from which students can draw appropriate and relevant information.</p> <p>Students will need to select and read material that contains relevant information. From this information they will need to identify accurately and compare the lines of reasoning and main points from the text and images. Students will then need to synthesise this information into a relevant form, eg for a presentation, discussion or an essay.</p> <p>Students may have to do some background reading for S1–S3.</p>
C3.3	<p>Write two different types of documents, each one giving different information about complex subjects.</p> <p>One document must be at least 1000 words long.</p>	<p>Students are required to produce two different types of document. At least one of these should be an extended document, for example a report or an essay of more than three pages.</p> <p>The document should have a form and style of writing which are fit both for its purpose and the complex subject matter covered. At least one of the documents should include an appropriate image that contains and effectively conveys relevant information. Specialist vocabulary should be used where appropriate and the information in the document should be clearly and coherently organised eg through the use of headings, paragraphs, etc.</p> <p>Students should ensure that the text is legible and that spelling, punctuation and grammar are accurate.</p>

Evidence

Candidate evidence for communication could include:

- tutor observation records
- preparatory notes
- audio/video tapes
- notes based on documents read
- essays.

Information and communication technology — Level 3

Show that you can plan and carry through a number of different tasks, one of which must be a major task covering ICT3.1, ICT3.2 and ICT3.3.

Each component, ICT3.1, ICT3.2 and ICT3.3, must be covered at least twice, and ICT3.3 must be covered for at least two different audiences. Smaller tasks may be used to ensure each component is covered.

Overall through at least two activities you must:

- include at least one ICT based information source
- include at least one non ICT based information source
- use at least one example of text, one example of image and one example of number
- use one example of combined information such as text and number, or image and number or text and image
- present evidence of purposeful use of email; one of these emails must have an attachment related to the task.

When producing work for their Advanced GCE in Mathematics, students will have numerous opportunities to use information technology. The Internet, CD ROMs, etc could be used to collect information, particularly when collecting data for use in the statistics units S1, S2 and S3. Documents can be produced using relevant software and images may be incorporated in those documents. Early drafts of documents could be emailed to tutors for initial comments and feedback.

For this Key Skill, students are required to carry out at least one 'substantial activity'. This is defined as 'an activity that includes a number of related tasks, where the results of one task will affect the carrying out of the others'. The activity should generate evidence for all three areas of evidence required in Part B of the ICT unit. If students undertaking coursework as part of their AS/A2 units in statistics use information technology, they will have opportunities to generate evidence for all three sections identified as part of a 'substantial activity'.

Mathematics students should utilise ICT as a modelling tool, particularly when using graphical calculators and spreadsheets. Accounts of their use in this way should be encouraged as part of students' ICT portfolio.

In addition, students will be able to use information technology to generate evidence for the Communication Key Skill. For example, the extended document with images, required for C3.3, could be generated using appropriate software.

Key skills portfolio evidence requirement		Opportunities for development or internal assessment
ICT3.1	Search for information, using different sources, and multiple search criteria in at least one case.	<p>Students will need to plan, and document, how they are to use IT as part of the activity, including how they will search for and incorporate relevant information from different electronic sources. These may include the Internet and CD ROM. Information selected must be relevant and of the appropriate quality.</p> <p>For example, opportunities for partially satisfying this criteria include:</p> <ul style="list-style-type: none"> ■ collecting data in secondary form from a variety of Internet sources including: <ol style="list-style-type: none"> (i) the DfEE website for educational performance tables (ii) the Office of Health Economics, etc.
ICT3.2	Enter and develop the information and derive new information.	<p>Students are required to bring together, in a consistent format, their selected information and use automated routines as appropriate.</p> <p>For example, opportunities for partially satisfying this criterion include:</p> <p>interrogating a professional database to extract relevant data for use in S1, S2 and S3</p> <p>recording data from an experiment in a spreadsheet and using its functions to calculate appropriate statistics and/or values, eg modelling a tide table to a cosine curve and calculating the error bounds of the model v actual data</p>
ICT3.3	Present combined information such as text with image, text with number, image with number.	<p>Students need to:</p> <ul style="list-style-type: none"> ■ develop a structure, which may involve the modification of templates, the application of page numbers, dates, etc. <p>Tutors may provide early feedback on layout on content and style that will result in formatting changes (early drafts should be kept as portfolio evidence).</p> <p>The final format should be suitable for its purpose and audience, eg AS coursework, ohts/handouts for a presentation, etc. The document should have accurate spelling (use of spell-checker) and have been proof-read</p>

Evidence

Candidate evidence for information technology could include:

- tutor observation records
- preparatory plans
- printouts with annotations
- draft documents.

Improving own learning and performance — Level 3

Provide at least **one** example of meeting the standard for LP3.1, LP3.2 and LP3.3 (the example should cover at least three targets). Overall, show you can use at least **two** different ways of learning to improve your performance.

Within Advanced GCE in Mathematics, students will have opportunities to develop and generate evidence, which meets part of the evidence requirement of this Key Skill.

To achieve this Key Skill students will need to carry out two study-based learning activities and two activity-based learning activities. Most of the units forming the Advanced GCE in Mathematics will provide opportunities for students to undertake study-based learning. Evidence for activity-based learning will depend on the approach adopted by the teacher. Fieldwork within the mechanics and statistics units readily lends itself to satisfying some of the criteria for LC3.1, LC3.2 and LC3.3.

One of the study-based learning activities must contain at least one complex task and periods of self-directed learning. Activities that generate evidence for this skill should take place over an extended period of time, eg three months. Over the period of the activity students should seek and receive feedback, from tutors and others, on their target setting and performance.

Key skills portfolio evidence requirement		Opportunities for development or internal assessment
LP3.1	Set targets using information from appropriate people and plan how these will be met.	Students plan how they are to produce their coursework. This will include setting realistic dates and targets and identification of potential problems and alternative courses of action. This will be determined with advice from others, eg their tutor. In mathematics the candidate could: <ul style="list-style-type: none"> ■ plan a rigorous timetable for home study, reviews and tutorials for all the units
LP3.2	Take responsibility for your learning, using your plan to help meet targets and improve your performance.	Students use the plan effectively when producing: <ul style="list-style-type: none"> ■ an extended account of a mechanics experiment/model. This will involve: <ul style="list-style-type: none"> ■ prioritising action ■ managing their time effectively; and ■ revising their plan as necessary. The candidate should: <ul style="list-style-type: none"> ■ seek and use feedback and support and draw on different approaches to learning as outlined in their detailed plan of action.
LP3.3	Review progress and establish evidence of your achievements.	Students should review their own progress and the quality of their learning and performance. They should identify targets met, providing evidence of achievements from relevant sources. They should identify with others, eg their tutor, action for improving their performance.

Evidence

Candidate evidence for improving own learning and performance could include:

- tutor records
- annotated action plans
- records of discussions
- learning log
- work produced.

Problem solving — Level 3

Provide at least **one** example of meeting the standard for PS3.1, PS3.2 and PS3.3. The example should include exploring at least **three** different ways of tackling a problem (for PS3.1).

For this Key Skill students are required to apply their problem solving skills to complex activities. They need to show that they can recognise, explore and describe problems, generate ways of solving problems, implement options and check whether the problem has been solved.

Key skills portfolio evidence requirement		Opportunities for development or internal assessment
PS3.1	Explore a problem and identify different ways of tackling it.	Students are required to select and use appropriate methods for generating different options for tackling the problem and compare the features of each option, selecting the most suitable one. Students need to be aware that there may be more than one method of solution of mathematical problems and be capable of comparing the effectiveness of each; comparing numerical methods to calculus methods for calculating the area under a curve in C1 and C2; comparing the effectiveness of grouping data when estimating the mean in S1: comparing various forms of statistical diagram for presenting data etc.
PS3.2	Plan and implement at least one way of solving the problem.	The implementation of the chosen option will need to be planned and permission gained to implement it. Implementation of the plan should involve full use of support and feedback from others with progress reviews and alterations to the plan as necessary.
PS3.3	Check if the problem has been solved and review your approach to problem solving.	On completion the outcomes need to be checked against the standards agreed at the start. The results of this should be recorded and the approach taken reviewed. For example: One complete run through a modelling cycle in mechanics, reviewing the model's effectiveness and redesigning the process to take account of any changes.

Evidence

Candidate evidence for problem solving could include:

- description of the problem
- tutor records and agreement of standards and approaches
- annotated action plans
- records of discussions
- descriptions of options
- records of reviews.

Working with others — Level 3

Provide at least **one** example of meeting the standard for WO3.1, WO3.2 and WO3.3, to include work in a group or team situation. You must check progress on two occasions (for WO3.2).

Key skills portfolio evidence requirement		Opportunities for development or internal assessment
WO3.1	Plan work with others.	<p>Students could work in groups of six to eight and be required to investigate a given topic. Initial work will require identification of and agreeing of objectives and planning how to meet these, including any necessary action and resources required. The group needs to agree responsibilities and working arrangements.</p> <p>For example, throughout the data collection activities in all statistics units, students should be encouraged to:</p> <ul style="list-style-type: none"> ■ discuss and agree on hypotheses to be tested ■ share out the data collection within the group, taking the opportunity to discuss relevant sampling techniques ■ effectively manage the time of each group member, agreeing targets and deadlines. <p>Partial satisfaction of these criteria relies on the teacher creating opportunities for data collection rather than allocating data that has already been prepared.</p>
WO3.2	Seek to develop co-operation and check progress towards your agreed objectives.	<p>For example, throughout the data collection activities in all statistics units, students should be encouraged to:</p> <ul style="list-style-type: none"> ■ discuss and agree on hypotheses to be tested ■ share out the data collection within the group, taking the opportunity to discuss relevant sampling techniques ■ effectively manage the time of each group member, agreeing targets and deadlines.
WO3.3	Review work with others and agree ways of improving collaborative work in the future.	<p>For example, throughout the data collection activities in all statistics units, students should be encouraged to:</p> <ul style="list-style-type: none"> ■ review outcomes against the agreed hypotheses ■ identify factors that have influenced the outcome ■ agree on the ways in which the activity could have been carried out more effectively.

Evidence

Candidate evidence for working with others could include:

- tutor observation records
- preparatory plans
- records of process and progress made
- evaluative reports.

Type of code	Use of code	Code number
National classification codes	Every specification is assigned to a national classification code indicating the subject area to which it belongs. Centres should be aware that students who enter for more than one GCE qualification with the same classification code will have only one grade (the highest) counted for the purpose of the school and college performance tables.	The classification codes for this specification are: 2210 Advanced Subsidiary GCE in Mathematics 2330 Advanced Subsidiary GCE in Further Mathematics 2230 Advanced Subsidiary GCE in Pure Mathematics 2230 Advanced Subsidiary GCE in Further Mathematics (Additional) 2210 Advanced GCE in Mathematics 2330 Advanced GCE in Further Mathematics 2230 Advanced GCE in Pure Mathematics 2230 Advanced GCE in Further Mathematics (Additional)
National Qualifications Framework (NQF) codes	Each qualification title is allocated a QCA National Qualifications Framework (NQF) code. The QCA National Qualifications Framework (NQF) code is known as a Qualification Accreditation Number (QAN). This is the code that features in the DfES Funding Schedule, Sections 96 and 97, and is to be used for all qualification funding purposes. The QCA QAN is the number that will appear on the candidate's final certification documentation.	The QANs for the qualifications in this publication are: 100/3412/2 Advanced GCE in Mathematics 100/3411/0 AS GCE in Mathematics
Unit codes	Each unit is assigned a unit code. This unit code is used as an entry code to indicate that a student wishes to take the assessment for that unit. Centres will only need to use the entry codes when entering their students.	Please see unit information
Cash in codes	The cash-in code is used as an entry code to aggregate the student's unit scores to obtain the overall grade for the qualification. Centres will only need to use the entry codes when entering their students.	Please see award information
Entry codes	The entry codes are used to: 1 enter a student for the assessment of a unit 2 aggregate the students' unit scores to obtain the overall grade for the qualification.	Please refer to the Edexcel Information Manual available on the Edexcel website.

Centres should be aware the only changes in the unit content between this and the previous specification are in units FP1, FP2, FP3, D1 and D2.

In 2009, examination opportunities will be given for both the new versions of these units and the legacy version of these units.

For students beginning these units in September 2008, the content of this specification needs to be used, alongside the updated versions of unit and cash-in codes.

For students who began their units in September 2007 or before need to complete Further Pure Mathematics or Decision Mathematics units from the previous specification (June 2004: UA014391)

Content for Core, Mechanics and Statistics Units has not changed.

Updated versions of the Formula Book and Statistical Tables will be available to centres in 2008.

Mapping documents from current Further Pure Mathematics and Decision Mathematics textbooks to the new content will be available from the Edexcel website.

Unit codes and information

The table below shows the legacy and updated unit codes.

Legacy units For teaching September 2007		Updated units For teaching September 2008	
Unit title	Unit code	Unit title	Unit code
Further Pure 1	6674	Further Pure 1	6667
Further Pure 2	6675	Further Pure 2	6668
Further Pure 3	6676	Further Pure 3	6669

Examination opportunities:

Session	Legacy units For teaching September 2007	Updated units For teaching September 2008
January 2009	FP1	FP1
June 2009	FP1, FP2, FP3	FP1, FP2, FP3
January 2010	FP1, FP2, FP3	FP1
June 2010 and all subsequent June sessions		FP1, FP2, FP3
January 2011 and all subsequent January sessions		FP1

AS/A levels often require assessment of a broad range of competences. This is because they are general qualifications and, as such, prepare candidates for a wide range of occupations and higher level courses.

The revised AS/A level qualification and subject criteria were reviewed to identify whether any of the competences required by the subject presented a potential barrier to any disabled candidates. If this was the case, the situation was reviewed again to ensure that such competences were included only where essential to the subject. The findings of this process were discussed with disability groups and with disabled people.

Reasonable adjustments are made for disabled candidates in order to enable them to access the assessments. For this reason, very few candidates will have a complete barrier to any part of the assessment. For information on reasonable adjustments please see the Edexcel website (www.edexcel.org.uk/sfc).

Candidates who are still unable to access a significant part of the assessment, even after exploring all possibilities through reasonable adjustments, may still be able to receive an award. They would be given a grade on the parts of the assessment they have taken and there would be an indication on their certificate that not all of the competencies have been addressed.

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We will inform centres of any changes to this issue. The latest issue can be found on the Edexcel website: www.edexcel.org.uk.

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